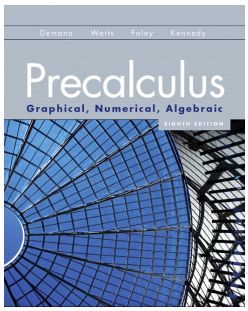
P.6
Solving Inequalities Algebraically and Graphically



What you'll learn about

- Solving Absolute Value Inequalities
- Solving Quadratic Inequalities
- Approximating Solutions to Inequalities
- Projectile Motion

… and why

These techniques are involved in using a graphing utility to solve inequalities in this textbook.

Solving Absolute Value Inequalities

Let u be an algebraic expression in x and let a be a real number with $a \geq 0$.

1. If $|u| < a$, then u is in the interval $(-a, a)$. That is,
$|u| < a$ if and only if $-a < u < a$.

2. If $|u| > a$, then u is in the interval $(-\infty, -a)$ or (a, ∞). That is,
$|u| > a$ if and only if $u < -a$ or $u > a$.

The inequalities $<$ and $>$ can be replaced with $\leq$ and $\geq$, respectively.

Example **Solving an Absolute Value Inequality**

Solve $|x + 3| < 5$.

Solution

Solve $|x + 3| < 5$.

$|x + 3| < 5$

$-5 < x + 3 < 5$

$-8 < x < 2$

As an interval the solution in $(-8, 2)$.

Example **Solving a Quadratic Inequality**

Solve $5x^2 - 11x \geq 12$.

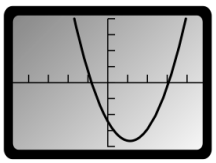
Solution

$5x^2 - 11x - 12 \geq 0$

$5x^2 - 11x - 12 = 0$

$(5x + 4)(x - 3) = 0$

$x = -\frac{4}{5}$ or $x = 3$



Use these solutions and a sketch of the equation $y = 5x^2 - 11x - 12$ to find the solution to the inequality in interval form $\left(-\infty, -\frac{4}{5}\right] \cup [3, \infty)$.

Projectile Motion

Suppose an object is launched vertically from a point s_0 feet above the ground with an initial velocity of v_0 feet per second. The vertical position s (in feet) of the object t seconds after it is launched is

$$s = -16t^2 + v_0 t + s_0.$$

Example **Finding Height of a Projectile**

A projectile is launched straight up from ground level with an initial velocity of 288ft/sec.

(a) When will the projectile's height above ground be 1152 ft?

(b) When will the projectile's height above ground be at least 1152 ft?

Solution

Here $s_0 = 0$ and $v_0 = 288$. So the projectile's height is $S = -16t^2 + 288t$.

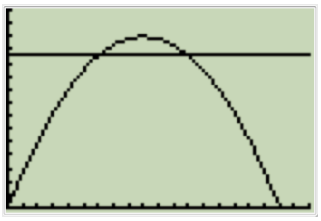
(a) Determine when $s = 1152$.

$$s = -16t^2 + 288t$$
$$1152 = -16t^2 + 288t$$
$$16t^2 - 288t + 1152 = 0$$
$$t^2 - 18t + 72 = 0$$
$$(t - 6)(t - 12) = 0$$
$$t = 6 \text{ or } t = 12$$

Determine when $s = 1152$.

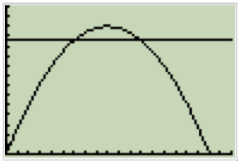
Solution continued

The projectile is 1152 ft above ground twice; the first time at $t = 6$ sec on the way up, and the second time at $t = 12$ sec on the way down.



Solution continued

(b) The projectile will be at least 1152 ft above ground when $s \geq 1152$. We can see from the figure together with the algebraic work in (a) that the solution is [6, 12]. This means that the projectile is at least 1152 ft above ground for times between $t = 6$ sec and $t = 12$ sec, including 6 and 12 sec.



Quick Review

Solve for x.

1. $-3 < 2x + 1 < 9$
2. $|2x + 1| = 3$
3. Factor completely. $4x^2 - 9$
4. Reduce the fraction to lowest terms. $\dfrac{x^2 - 49}{x^2 + 7x}$
5. Add the fractions and simplify. $\dfrac{x}{x+1} + \dfrac{x+2}{x}$

Quick Review Solutions

Solve for x.

1. $-3 < 2x + 1 < 9$ $-2 < x < 4$
2. $|2x + 1| = 3$ $x = -2$ or $x = 1$
3. Factor completely. $4x^2 - 9$ $(2x - 3)(2x + 3)$
4. Reduce the fraction to lowest terms. $\dfrac{x^2 - 49}{x^2 + 7x}$ $\dfrac{x-7}{x}$
5. Add the fractions and simplify. $\dfrac{x}{x+1} + \dfrac{x+2}{x}$ $\dfrac{2x^2 + 3x + 2}{x^2 + x}$

Chapter Test

1. Write the number in scientific notation.
The diameter of a red blood corpuscle is about 0.000007 meter.
2. Find the standard form equation for the circle with center $(5, -3)$ and radius 4.
3. Find the slope of the line through the points $(-1, -2)$ and $(4, -5)$.
4. Find the equation of the line through $(2, -3)$ and perpendicular to the line $2x + 5y = 3$.
5. Solve the equation algebraically. $\dfrac{x-2}{3} + \dfrac{x+5}{2} = \dfrac{1}{3}$
6. Solve the equation algebraically. $6x^2 + 7x = 3$

Chapter Test

7. Solve the equation algebraically. $|4x + 1| = 3$
8. Solve the inequality. $|3x + 4| \geq 2$
9. Solve the inequality. $4x^2 + 12x + 9 \geq 0$
10. Perform the indicated operation, and write the result in standard form. $(5 - 7i) - (3 - 2i)$

Chapter Test Solutions

1. Write the number in scientific notation.
The diameter of a red blood corpuscle is about 0.000007 meter. 7×10^{-6}
2. Find the standard form equation for the circle with center $(5, -3)$ and radius 4. $(x-5)^2 + (y+3)^2 = 16$
3. Find the slope of the line through the points $(-1, -2)$ and $(4, -5)$. $-\dfrac{3}{5}$
4. Find the equation of the line through $(2, -3)$ and perpendicular to the line $2x + 5y = 3$. $y = \dfrac{5}{2}x - 8$
5. Solve the equation algebraically. $\dfrac{x-2}{3} + \dfrac{x+5}{2} = \dfrac{1}{3}$ $x = -\dfrac{9}{5}$
6. Solve the equation algebraically. $6x^2 + 7x = 3$ $x = \dfrac{1}{3}$ or $x = -\dfrac{3}{2}$

Chapter Test Solutions

7. Solve the equation algebraically. $|4x + 1| = 3$ $x = \dfrac{1}{2}$ or $x = -1$
8. Solve the inequality. $|3x + 4| \geq 2$ $(-\infty, -2] \cup \left[-\dfrac{2}{3}, \infty\right)$
9. Solve the inequality. $4x^2 + 12x + 9 \geq 0$ $(-\infty, \infty)$
10. Perform the indicated operation, and write the result in standard form. $(5 - 7i) - (3 - 2i)$ $2 - 5i$