LESSON 3.8

The Centroid

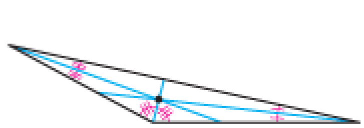
The universe may be as great as they say, but it wouldn't be missed if it didn't exist.

PIET HEIN

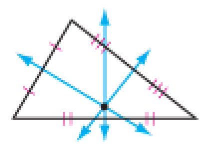
In the previous lesson you discovered that the three angle bisectors are concurrent, the three perpendicular bisectors of the sides are concurrent, and the three altitudes in a triangle are concurrent. You also discovered the properties of the incenter and the circumcenter. In this lesson you will investigate the medians of a triangle.

You may choose to do the first investigation using the **Dynamic Geometry Exploration** The Centroid at www.keymath.com/DG.

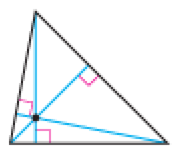
keymath.com/DG



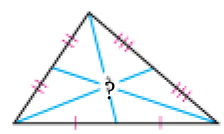
Three angle bisectors (incenter)



Three perpendicular bisectors (circumcenter)



Three altitudes (orthocenter)



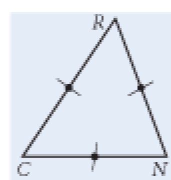
Three medians ?

Investigation 1
Are Medians Concurrent?

You will need

- construction tools
- geometry software (optional)

Each person in your group should draw a different triangle for this investigation. Make sure you have at least one acute triangle, one obtuse triangle, and one right triangle in your group.

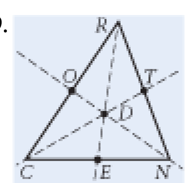


Step 1 On a sheet of patty paper, draw as large a scalene triangle as possible and label it *CNR*, as shown at right. Locate the midpoints of the three sides. Construct the medians and complete the conjecture.

Median Concurrency Conjecture C-14

The three medians of a triangle __?__.

The point of concurrency of the three medians is the **centroid.**

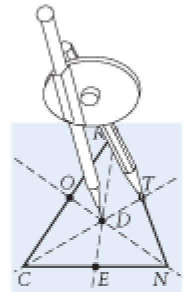


Step 2 Label the three medians $\overline{CT}$, $\overline{NO}$, and $\overline{RE}$. Label the centroid *D*.

Step 3 Use your compass or another sheet of patty paper to investigate whether there is anything special about the centroid. Is the centroid equidistant from the three vertices? From the three sides? Is the centroid the midpoint of each median?

Step 4 The centroid divides a median into two segments. Focus on one median. Use your patty paper or compass to compare the length of the longer segment to the length of the shorter segment and find the ratio.

Step 5 Find the ratios of the lengths of the segment parts for the other two medians. Do you get the same ratio for each median?

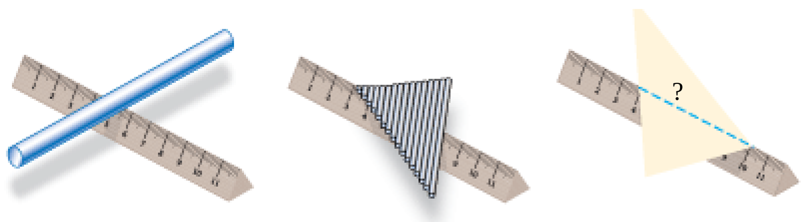


Compare your results with the results of others. State your discovery as a conjecture, and add it to your conjecture list.

Centroid Conjecture C-15

The centroid of a triangle divides each median into two parts so that the distance from the centroid to the vertex is ___?___ the distance from the centroid to the midpoint of the opposite side.

In earlier lessons you discovered that the midpoint of a segment is the balance point or center of gravity. You also saw that when a set of segments is arranged into a triangle, the line through each midpoint of a side and the opposite vertex can act as a line of balance for the triangle. Can you then balance a triangle on a median? Let's take a look.



Investigation 2
Balancing Act

You will need
- cardboard
- a straightedge

Use your patty paper from Investigation 1 for this investigation. If you used geometry software, print out your triangle with medians.

Step 1 Place your patty paper or printout from the previous investigation on a piece of mat board or cardboard. With a sharp pencil tip or compass tip, mark the three vertices, the three midpoints, and the centroid on the board.

Step 2 Draw the triangle and medians on the cardboard. Cut out the cardboard triangle.

Step 3 Try balancing the triangle on one of the three medians by placing the median on the edge of a ruler. If you are successful, what does that imply about the areas of the two triangles formed by one median? Try balancing the triangle on another median. Will it balance on each of the three medians?

Step 4 Is there a single point where you can balance the triangle?

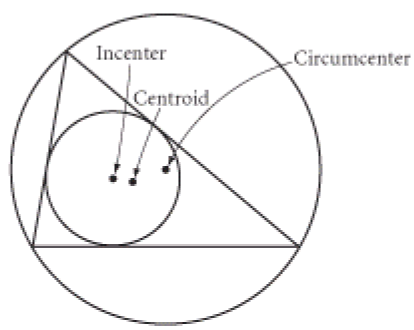
If you have found the balancing point for the triangle, you have found its **center of gravity.** State your discovery as a conjecture, and add it to your conjecture list.

Center of Gravity Conjecture C-16

The _?_ of a triangle is the center of gravity of the triangular region.

The triangle balances on each median and the centroid is on each median, so the triangle balances on the centroid. As long as the weight of the cardboard is distributed evenly throughout the triangle, you can balance any triangle at its centroid. For this reason, the centroid is a very useful point of concurrency, especially in physics.

You have discovered special properties of three of the four points of concurrency—the incenter, the circumcenter, and the centroid. The incenter is the center of an inscribed circle, the circumcenter is the center of a circumscribed circle, and the centroid is the center of gravity.



You can learn more about the orthocenter in the project Is There More to the Orthocenter?

Science CONNECTION

In physics, the center of gravity of an object is an imaginary point where the total weight is concentrated. The center of gravity of a tennis ball, for example, would be in the hollow part, not in the actual material of the ball. The idea is useful in designing structures as complicated as bridges or as simple as furniture. Where is the center of gravity of the human body?

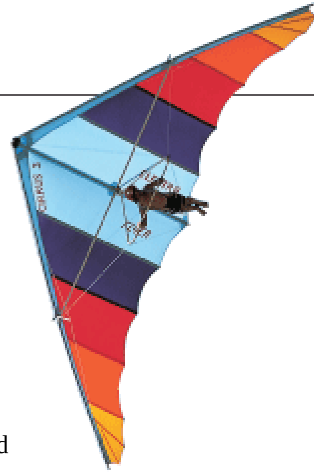
EXERCISES

Keymath.com Practice Your Skills

You will need

Construction tools for Exercises 5 and 6

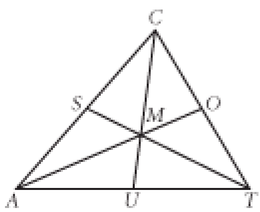
Geometry software for Exercise 7



1. Birdy McFly is designing a large triangular hang glider. She needs to locate the center of gravity for her glider. Which point does she need to locate? Birdy wishes to decorate her glider with the largest possible circle within her large triangular hang glider. Which point of concurrency does she need to locate?

In Exercises 2–4, use your new conjectures to find each length.

2. Point M is the centroid. (h)



$CM = 16$

$MO = 10$

$TS = 21$

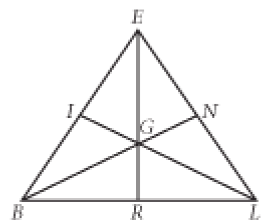
$AM = \underline{?}$

$SM = \underline{?}$

$TM = \underline{?}$

$UM = \underline{?}$

3. Point G is the centroid.



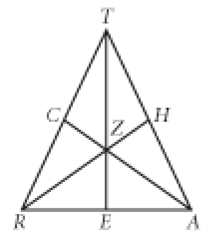
$GI = GR = GN$

$ER = 36$

$BG = \underline{?}$

$IG = \underline{?}$

4. Point Z is the centroid.



$CZ = 14$

$TZ = 30$

$RZ = AZ$

$RH = \underline{?}$

$TE = \underline{?}$

5. ***Construction*** Construct an equilateral triangle, then construct angle bisectors from two vertices, medians from two vertices, and altitudes from two vertices. What can you conclude?

6. ***Construction*** On patty paper, draw a large isosceles triangle with an acute vertex angle that measures less than 40°. Copy it onto three other pieces of patty paper. Construct the centroid on one patty paper, the incenter on a second, the circumcenter on a third, and the orthocenter on a fourth. Record the results of all four pieces of patty paper on one piece of patty paper. What do you notice about the four points of concurrency? What is the order of the four points of concurrency from the vertex to the opposite side in an acute isosceles triangle?

7. ***Technology*** Use geometry software to construct a large isosceles acute triangle. Construct the four points of concurrency. Hide all constructions except for the points of concurrency. Label them. Drag until it has an obtuse vertex angle. Now what is the order of the four points of concurrency from the vertex angle to the opposite side? When did the order change? Do the four points ever become one?

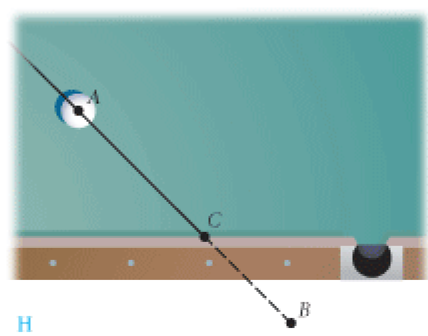
8. ***Mini-Investigation*** Where do you think the center of gravity is located on a square? A rectangle? A rhombus? In each case the center of gravity is not that difficult to find, but what about an ordinary quadrilateral? Experiment to discover a method for finding the center of gravity for a quadrilateral by geometric construction. Test your method on a large cardboard quadrilateral. ⓗ

Review

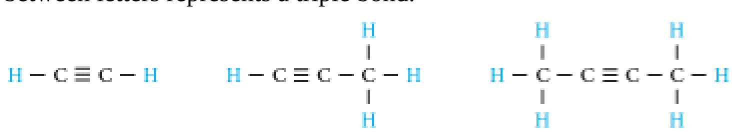
9. Sally Solar is the director of Lunar Planning for Galileo Station on the moon. She has been asked to locate the new food production facility so that it is equidistant from the three main lunar housing developments. Which point of concurrency does she need to locate?

10. Construct circle *O*. Place an arbitrary point *P* within the circle. Construct the longest chord passing through *P*. Construct the shortest chord passing through *P*. How are they related?

11. A billiard ball is hit so that it travels a distance equal to *AB* but bounces off the cushion at point *C*. Copy the figure, and sketch where the ball will rest.



12. ***Application*** In alkyne molecules all the bonds are single bonds except one triple bond between two carbon atoms. The first three alkynes are modeled below. The dash (–) between letters represents single bonds. The triple dash (≡) between letters represents a triple bond.

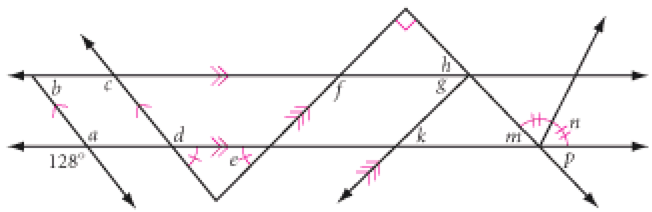


Ethyne (C_2H_2) Propyne (C_3H_4) Butyne (C_4H_6)

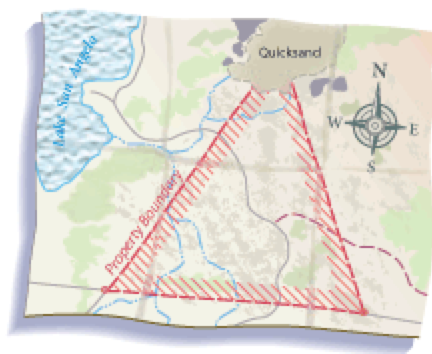
Sketch the alkyne with eight carbons in the chain. What is the general rule for alkynes ($C_nH_?$)? In other words, if there are *n* carbon atoms (C), how many hydrogen atoms (H) are in the alkyne?

13. When plane figure A is rotated about the line, it produces the solid figure B. What is the plane figure that produces the solid figure D?

14. Copy the diagram below. Use your Vertical Angles Conjecture and Parallel Lines Conjecture to calculate each lettered angle measure.



15. A brother and a sister have inherited a large triangular plot of land. The will states that the property is to be divided along the altitude from the northernmost point of the property. However, the property is covered with quicksand at the northern vertex. The will states that the heir who figures out how to draw the altitude without using the northern vertex point gets to choose his or her parcel first. How can the heirs construct the altitude? Is this a fair way to divide the land? Why or why not? ⓗ



16. At the college dorm open house, each of the 20 dorm members invites two guests. How many greetings are possible if you do not count dorm members greeting each other? ⓗ

IMPROVING YOUR REASONING SKILLS

The Dealer's Dilemma

In the game of bridge, the dealer deals 52 cards in a clockwise direction among four players. You are playing a game in which you are the dealer. You deal the cards, starting with the player on your left. However, in the middle of dealing, you stop to answer the phone. When you return, no one can remember where the last card was dealt. (And, of course, no cards have been touched.) Without counting the number of cards in anyone's hand or the number of cards yet to be dealt, how can you rapidly finish dealing, giving each player exactly the same cards she or he would have received if you hadn't been interrupted?

Exploration

In the previous lessons you discovered the four points of concurrency: circumcenter, incenter, orthocenter, and centroid. In this activity you will discover how these points relate to a special line, the **Euler line.**

The Euler line is named after the Swiss mathematician Leonhard Euler (1707–1783), who proved that three points of concurrency are collinear.

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You may choose to do this activity using the **Dynamic Geometry Exploration** The Euler Line at www.keymath.com/DG

Activity

Three Out of Four

You will need

- patty paper
- geometry software (optional)

You are going to look for a relationship among the points of concurrency.

Step 1 Draw a scalene triangle and have each person in your group trace the same triangle on a separate piece of patty paper.

Step 2 Have each group member construct with patty paper a different point of the four points of concurrency for the triangle.

Step 3 Record the group's results by tracing and labeling all four points of concurrency on one of the four pieces of patty paper. What do you notice? Compare your group results with the results of other groups near you. State your discovery as a conjecture.

Euler Line Conjecture

The _?_, _?_, and _?_ are the three points of concurrency that always lie on a line.

The three special points that lie on the Euler line determine a segment called the **Euler segment.** The point of concurrency between the two endpoints of the Euler segment divides the segment into two smaller segments whose lengths have an exact ratio.

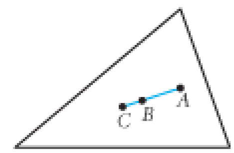
Step 4 With a compass or patty paper, compare the lengths of the two parts of the Euler segment. What is the ratio? Compare your group's results with the results of other groups and state your conjecture.

Euler Segment Conjecture

The ? divides the Euler segment into two parts so that the smaller part is ? the larger part.

Step 5 Use your conjectures to solve this problem.

$\overline{AC}$ is an Euler segment containing three points of concurrency, $A, B, C,$ so that $AB > BC$. $AC = 24$ m. $AB =$?. $BC =$?.



project

IS THERE MORE TO THE ORTHOCENTER?

At this point you may still wonder what's special about the orthocenter. It does lie on the Euler line. Is there anything else surprising or special about it?

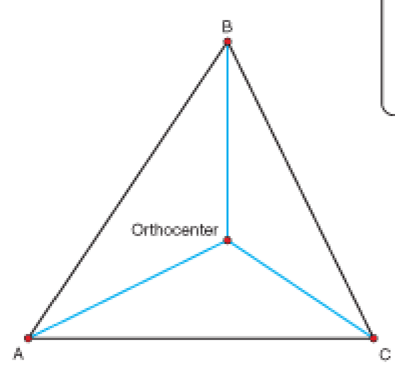
Use geometry software to investigate the orthocenter. Construct a triangle *ABC* and its orthocenter *O*, Drag a vertex of the triangle around. Where does the orthocenter lie in an acute triangle? An obtuse triangle? A right triangle?

Hide the altitudes. Draw segments from each vertex to the orthocenter shown. Now find the orthocenter of each of the three new triangles formed. What happens?

Experiment dragging the different points, and observe the relationships among the four orthocenters.

Write a paragraph about your findings, concluding with a conjecture about the orthocenter. Your project should include

- Your observations about the orthocenter, with drawings.
- Answers to all the questions above.
- A conjecture stating what's special about the orthocenter.



THE GEOMETER'S SKETCHPAD

The Geometer's Sketchpad was used to create this diagram and to hide the unnecessary lines. Using Sketchpad, you can quickly construct triangles and their points of concurrency. Once you make a conjecture, you can drag to change the shape of the triangle to see whether your conjecture is true.