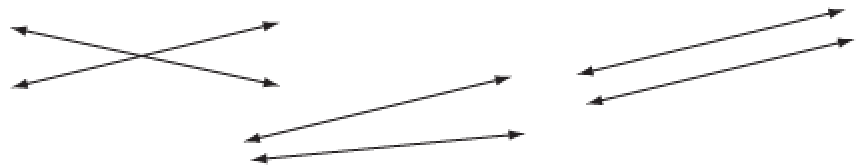
LESSON 3.5

Constructing Parallel Lines

When you stop to think, don't forget to start up again.

ANONYMOUS

Parallel lines are lines that lie in the same plane and do not intersect.



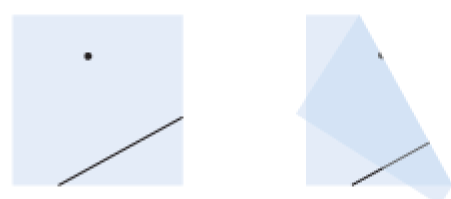
The lines in the first pair shown above intersect. They are clearly not parallel. The lines in the second pair do not meet as drawn. However, if they were extended, they would intersect. Therefore, they are not parallel. The lines in the third pair appear to be parallel, but if you extend them far enough in both directions, can you be sure they won't meet? There are many ways to be sure that the lines are parallel.

Investigation
Constructing Parallel Lines by Folding

You will need

- patty paper
- a straightedge

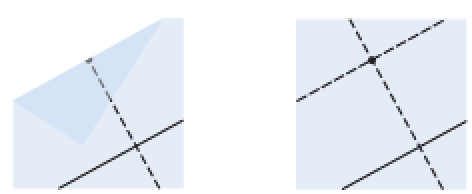
How would you check whether two lines are parallel? One way is to draw a transversal and compare corresponding angles. You can also use this idea to *construct* a pair of parallel lines.



Step 1   Step 2

Step 1 Draw a line and a point on patty paper as shown.

Step 2 Fold the paper to construct a perpendicular so that the crease runs through the point as shown. Describe the four newly formed angles.



Step 3   Step 4

Step 3 Through the point, make another fold that is perpendicular to the first crease.

Step 4 Compare the pairs of corresponding angles created by the folds. Are they all congruent? Why? What conclusion can you make about the lines?

There are many ways to construct parallel lines. You can construct parallel lines much more quickly with patty paper than with compass and straightedge. You can also use properties you discovered in the Parallel Lines Conjecture to construct parallel lines by duplicating corresponding angles, alternate interior angles, or alternate exterior angles. Or you can construct two perpendiculars to the same line. In the exercises you will practice all of these methods.

EXERCISES

You will need

Construction tools for Exercises 1–9

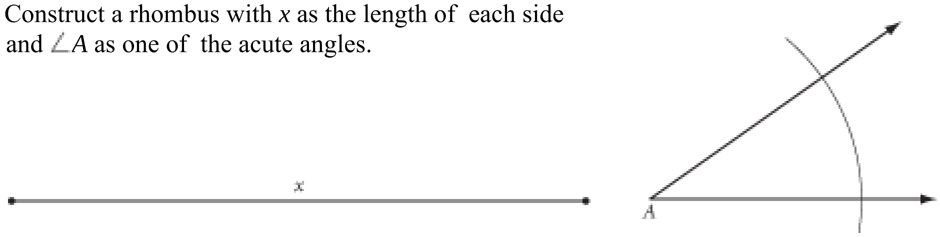
Construction In Exercises 1–9, use the specified construction tools to do each construction. If no tools are specified, you may choose either patty paper or compass and straightedge.

1. Use compass and straightedge. Draw a line and a point not on the line. Construct a second line through the point that is parallel to the first line, by duplicating alternate interior angles.

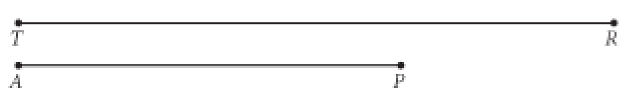
2. Use compass and straightedge. Draw a line and a point not on the line. Construct a second line through the point that is parallel to the first line, by duplicating corresponding angles.

3. Construct a square with perimeter *z*. (h)

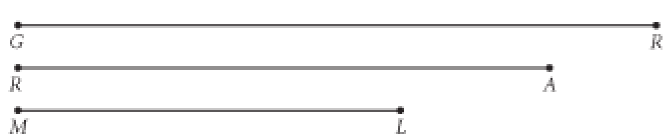
4. Construct a rhombus with *x* as the length of each side and $\angle A$ as one of the acute angles.



5. Construct trapezoid *TRAP* with $\overline{TR}$ and $\overline{AP}$ as the two parallel sides and with *AP* as the distance between them. (There are many solutions!)

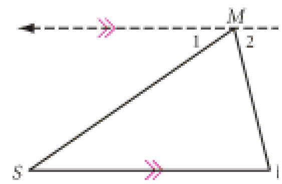


6. Using patty paper and straightedge, or a compass and straightedge, construct parallelogram *GRAM* with $\overline{RG}$ and $\overline{RA}$ as two consecutive sides and *ML* as the distance between $\overline{RG}$ and $\overline{AM}$. (How many solutions can you find?)



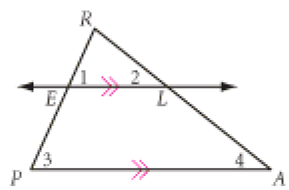
You may choose to do the mini-investigations in Exercises 7, 9, and 11 using geometry software.

7. ***Mini-Investigation*** Draw a large scalene acute triangle and label it $\triangle SUM$. Through vertex M construct a line parallel to side $\overline{SU}$ as shown in the diagram. Use your protractor or a piece of patty paper to compare $\angle 1$ and $\angle 2$ with the other two angles of the triangle ($\angle S$ and $\angle U$). Notice anything special? Write down what you observe.



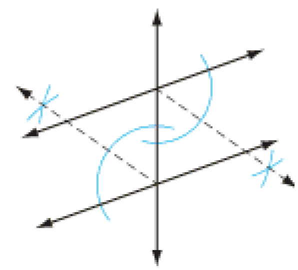
8. ***Developing Proof*** Use deductive reasoning to explain why your observation in Exercise 7 is true for any triangle.

9. ***Mini-Investigation*** Draw a large scalene acute triangle and label it $\triangle PAR$. Place point E anywhere on side PR, and construct a line $\overleftrightarrow{EL}$ parallel to side $\overline{PA}$ as shown in the diagram. Use your ruler to measure the lengths of the four segments $\overline{AL}$, $\overline{LR}$, $\overline{RE}$, and $\overline{EP}$, and compare ratios $\frac{RL}{LA}$ and $\frac{RE}{EP}$. Notice anything special? Write down what you observe.



10. ***Developing Proof*** Measure the four labeled angles in Exercise 9. Notice anything special? Use deductive reasoning to explain why your observation is true for any triangle.

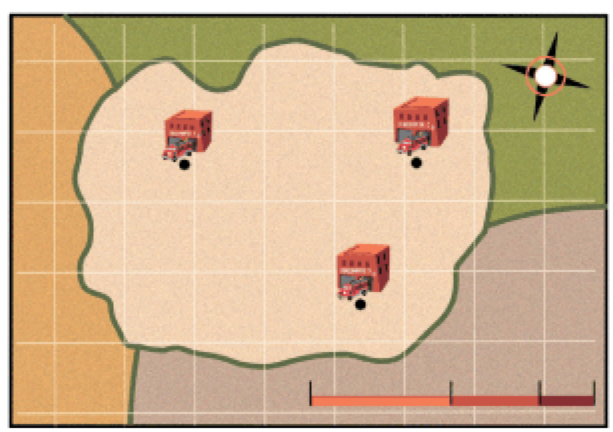
11. ***Mini-Investigation*** Draw a pair of parallel lines by tracing along both edges of your ruler. Draw a transversal. Use your compass to bisect each angle of a pair of alternate interior angles. What shape is formed?



12. ***Developing Proof*** Use deductive reasoning to explain why the resulting shape is formed in Exercise 11.

Review

13. There are three fire stations in the small county of Dry Lake. County planners need to divide the county into three zones so that fire alarms alert the closest station. Trace the county and the three fire stations onto patty paper, and locate the boundaries of the three zones. Explain how these boundaries solve the problem. ⓗ



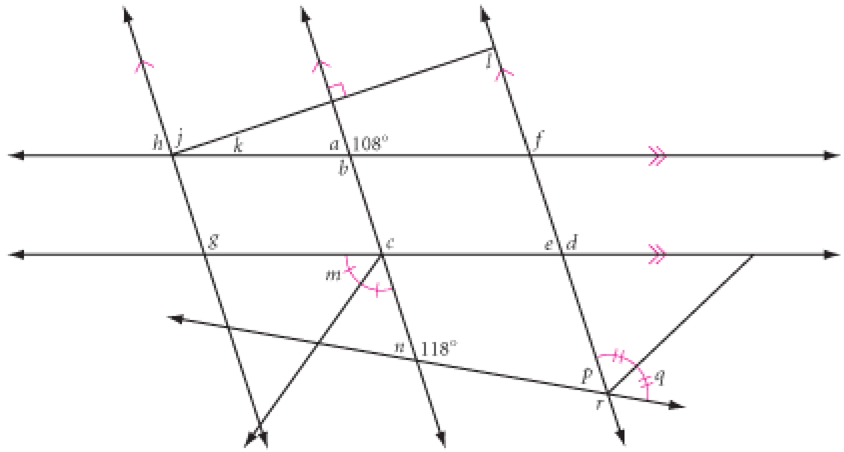
Sketch or draw each figure in Exercises 14–16. Label the vertices with the appropriate letters. Use the special marks that indicate right angles, parallel segments, and congruent segments and angles.

14. Sketch trapezoid $ZOID$ with $\overline{ZO} \parallel \overline{ID}$, point T the midpoint of $\overline{OI}$, and R the midpoint of $\overline{ZD}$. Sketch segment TR.

15. Draw rhombus $ROMB$ with $m\angle R = 60°$ and diagonal $\overline{OB}$.

16. Draw rectangle $RECK$ with diagonals $\overline{RC}$ and $\overline{EK}$ both 8 cm long and intersecting at point W.

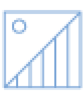
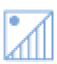
17. ***Developing Proof*** Copy the diagram below. Use your conjectures to calculate the measure of each lettered angle. Explain how you determined measures *m*, *p*, and *r*.



IMPROVING YOUR VISUAL THINKING SKILLS

Visual Analogies

Which of the designs at right complete the statements at left? Explain.

1.  is to  as  is to ?

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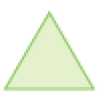
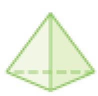
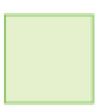
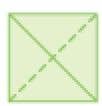
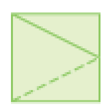
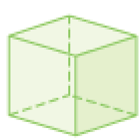
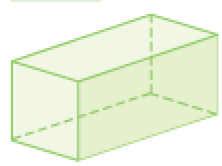
2.  is to  as  is to ?

A.  B.  C.  D. 

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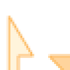
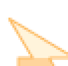
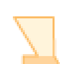
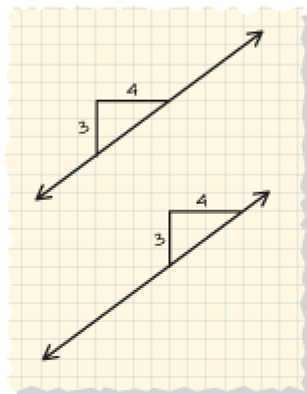
A.  B.  C.  D. 

USING YOUR ALGEBRA SKILLS 3

Slopes of Parallel and Perpendicular Lines



If two lines are parallel, how do their slopes compare? If two lines are perpendicular, how do *their* slopes compare? In this lesson you will review properties of the slopes of parallel and perpendicular lines.

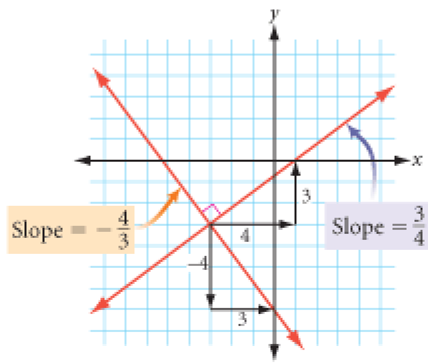
If the slopes of two or more distinct lines are equal, are the lines parallel? To find out, try drawing on graph paper two lines that have the same slope triangle.

Yes, the lines are parallel. In fact, in coordinate geometry, this is the definition of parallel lines. The converse of this is true as well: If two lines are parallel, their slopes must be equal.

Parallel Slope Property

In a coordinate plane, two distinct lines are parallel if and only if their slopes are equal, or they are both vertical lines.

If two lines are perpendicular, their slope triangles have a different relationship. Study the slopes of the two perpendicular lines at right.



Perpendicular Slope Property

In a coordinate plane, two nonvertical lines are perpendicular if and only if their slopes are opposite reciprocals of each other.

Can you explain why the slopes of perpendicular lines would have opposite signs? Can you explain why they would be reciprocals? Why do the lines need to be nonvertical?

EXAMPLE A

Consider $A(-15,-6)$, $B(6, 8)$, $C(4,-2)$ and $D(-4, 10)$. Are $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ parallel, perpendicular, or neither?

▶ Solution

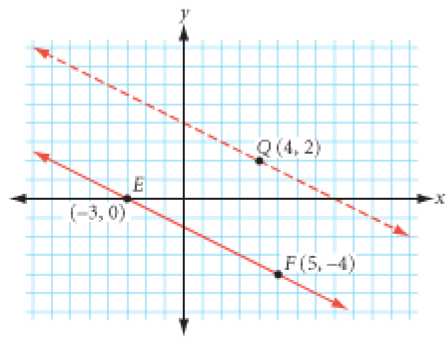
Calculate the slope of each line.

$$\text{slope of } \overleftrightarrow{AB} = \frac{8-(-6)}{6-(-15)} = \frac{2}{3} \qquad \text{slope of } \overleftrightarrow{CD} = \frac{10-(-2)}{-4-4} = -\frac{3}{2}$$

The slopes, $\frac{2}{3}$ and $-\frac{3}{2}$, are opposite reciprocals of each other, so $\overleftrightarrow{AB} \perp \overleftrightarrow{CD}$..

EXAMPLE B

Given points $E(-3, 0)$, $F(5,-4)$, and $Q(4, 2)$, find the coordinates of a point P such that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{EF}$.



▶ Solution

We know that if $\overleftrightarrow{PQ} \parallel \overleftrightarrow{EF}$, then the slope of $\overleftrightarrow{PQ}$ equals the slope of $\overleftrightarrow{EF}$. First find the slope of $\overleftrightarrow{EF}$.

$$\text{slope of } \overleftrightarrow{EF} = \frac{-4-0}{5-(-3)} = \frac{-4}{8} = -\frac{1}{2}$$

There are many possible ordered pairs (x, y) for P. Use (x, y) as the coordinates of P, and the given coordinates of Q, in the slope formula to get

$$\frac{2-y}{4-x} = -\frac{1}{2}$$

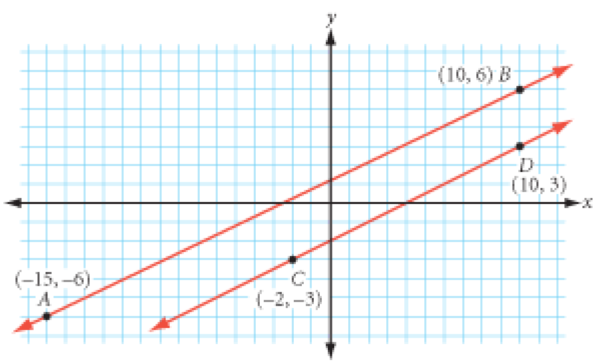
Now you can treat the denominators and numerators as separate equations.

$$4 - x = 2 \qquad 2 - y = -1$$
$$-x = -2 \qquad -y = -3$$
$$x = 2 \qquad y = 3$$

Thus one possibility is $P(2, 3)$. How could you find another ordered pair for P? Here's a hint: How many different ways can you express $-\frac{1}{2}$?

Language CONNECTION

Coordinate geometry is sometimes called "analytic geometry." This term implies that you can use algebra to further analyze what you see. For example, consider $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$. They look parallel, but looks can be deceiving. Only by calculating the slopes will you see that the lines are not truly parallel.



EXERCISES

For Exercises 1–4, determine whether each pair of lines through the points given below is parallel, perpendicular, or neither.

$A(1, 2)$ $B(3, 4)$ $C(5, 2)$ $D(8, 3)$ $E(3, 8)$ $F(-6, 5)$

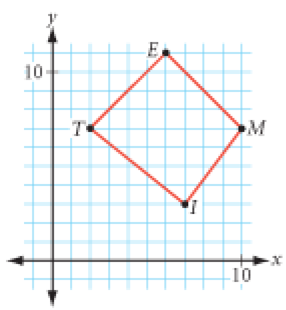
1. $\overleftrightarrow{AB}$ and $\overleftrightarrow{BC}$ **2.** $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ **3.** $\overleftrightarrow{AB}$ and $\overleftrightarrow{DE}$ **4.** $\overleftrightarrow{CD}$ and $\overleftrightarrow{EF}$

5. Given $A(0, -3)$, $B(5, 3)$, and $Q(-3, -1)$, find two possible locations for a point P such that $\overleftrightarrow{PQ}$ is parallel to $\overleftrightarrow{AB}$.

6. Given $C(-2, -1)$, $D(5, -4)$, and $Q(4, 2)$, find two possible locations for a point P such that $\overleftrightarrow{PQ}$ is perpendicular to $\overleftrightarrow{CD}$.

For Exercises 7–9, find the slope of each side, and then determine whether each figure is a trapezoid, a parallelogram, a rectangle, or just an ordinary quadrilateral. Explain how you know.

7.



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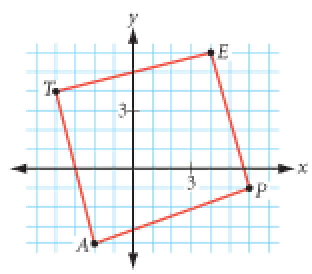

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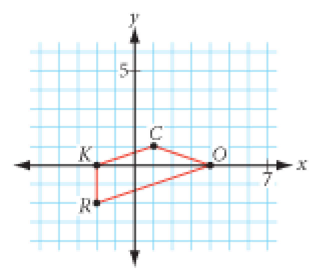

10. Quadrilateral *HAND* has vertices $H(-5, -1)$, $A(7, 1)$, $N(6, 7)$, and $D(-6,5)$.

a. Is quadrilateral *HAND* a parallelogram? A rectangle? Neither? Explain how you know.

b. Find the midpoint of each diagonal. What can you conjecture?

11. Quadrilateral *OVER* has vertices $O(-4, 2)$, $V(1, 1)$, $E(0, 6)$, and $R(-5, 7)$.

a. Are the diagonals perpendicular? Explain how you know.

b. Find the midpoint of each diagonal. What can you conjecture?

c. What type of quadrilateral does *OVER* appear to be? Explain how you know.

12. Consider the points $A(-5, -2)$, $B(1, 1)$, $C(-1, 0)$, and $D(3, 2)$.

a. Find the slopes of $\overline{AB}$ and $\overline{CD}$.

b. Despite their slopes, $\overline{AB}$ and $\overline{CD}$ are not parallel. Why not?

c. What word in the Parallel Slope Property addresses the problem in 12b?

13. Given $A(-3, 2)$, $B(1, 5)$, and $C(7, -3)$, find point D such that quadrilateral *ABCD* is a rectangle.