



LESSON

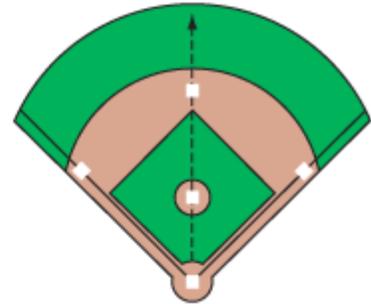
3.4

Constructing Angle Bisectors

Challenges make you discover things about yourself that you never really knew.

CICELY TYSON

On a softball field, the pitcher's mound is the same distance from each foul line, so it lies on the angle bisector of the angle formed by the foul lines. As with a perpendicular bisector of a segment, an angle bisector forms a line of symmetry. While the definition in Chapter 1 defined an angle bisector as a ray, you may also refer to a segment as an angle bisector if it lies on the ray and passes through the vertex.



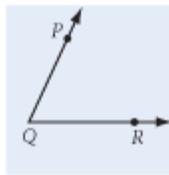
Investigation 1

Angle Bisecting by Folding

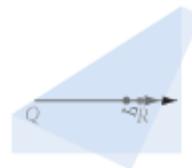
You will need

- patty paper
- a straightedge

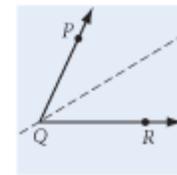
Each person should draw his or her own acute angle for this investigation.



Step 1



Step 2



Step 3

- Step 1 On patty paper, draw a large-scale angle. Label it PQR .
- Step 2 Fold your patty paper so that \overline{QP} and \overline{QR} coincide. Crease the fold.
- Step 3 Unfold your patty paper. Draw a ray with endpoint Q along the crease. Does the ray bisect $\angle PQR$? How can you tell?
- Step 4 Repeat Steps 1–3 with an obtuse angle. Do you use different methods for finding the bisectors of different kinds of angles?

- Step 5 Place a point on your angle bisector. Label it A . Compare the distances from A to each of the two sides. Remember that "distance" means *shortest* distance! Try it with other points on the angle bisector. Compare your results with those of others. Copy and complete the conjecture.

Angle Bisector Conjecture

C-8

If a point is on the bisector of an angle, then it is ? from the sides of the angle.

You've found the bisector of an angle by folding patty paper. Now let's see how you can construct the angle bisector with a compass and a straightedge.



Investigation 2 Angle Bisecting with Compass

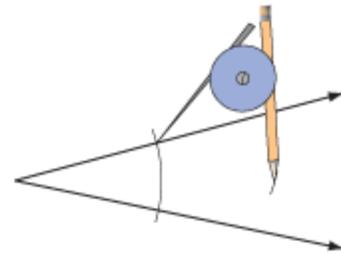
You will need

- a compass
- a straightedge

In this investigation, you will find a method for bisecting an angle using a compass and straightedge. Each person in your group should investigate a different angle.

- | | |
|--------|--|
| Step 1 | Draw an angle. |
| Step 2 | Find a method for constructing the bisector of the angle. Experiment!

Hint: Start by drawing an arc centered at the vertex. |
| Step 3 | Once you think you have constructed the angle bisector, fold your paper to see if the ray you constructed is actually the bisector. Share your method with other students in your group. Agree on a best method. |
| Step 4 | Write a summary of what you did in this investigation. |



In earlier lessons, you learned to construct a 90° angle. Now you know how to bisect an angle. What angles can you construct by combining these two skills?



EXERCISES

You will need

Construction For Exercises 1–5, match each geometric construction with its diagram.

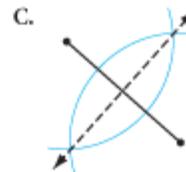
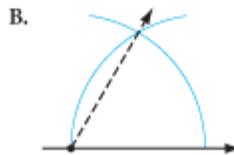
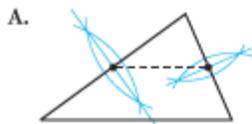


Construction tools for Exercises 1–12



Geometry software for Exercise 22

- | | |
|--------------------------------------|---|
| 1. Construction of an angle bisector | 2. Construction of a median |
| 3. Construction of a midsegment | 4. Construction of a perpendicular bisector |
| | 5. Construction of an altitude |



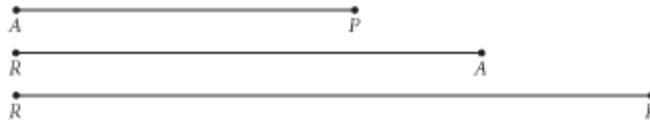
Construction For Exercises 6–12, construct a figure with the given specifications.

6. Given:



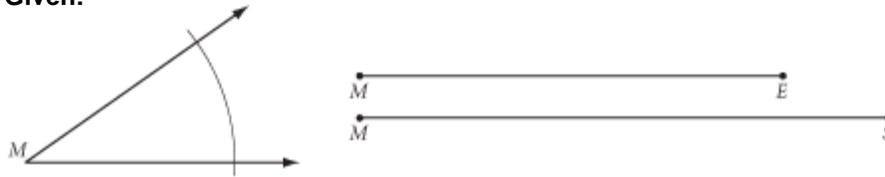
Construct: An isosceles right triangle with z as the length of each of the two congruent sides

7. Given:



Construct: $\triangle RAP$ with median \overline{PM} and angle bisector \overline{RB}

8. Given:



Construct: $\triangle MSE$ with \overline{OU} , where O is the midpoint of \overline{MS} and U is the midpoint of \overline{SE}

9. Construct an angle with each given measure and label it. Remember, you may use only your compass and straightedge. No protractor!
- a. 90° b. 45° c. 135°

10. Draw a large acute triangle. Bisect the angle at one vertex with a compass and a straightedge. Construct an altitude from the second vertex and a median from the third vertex.

11. Repeat Exercise 10 with patty paper. Which set of construction tools do you prefer? Why?

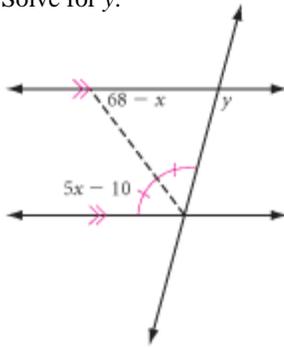
12. Use your straightedge to construct a linear pair of angles. Use your compass to bisect each angle of the linear pair. What do you notice about the two angle bisectors? Can you make a conjecture? Can you explain why it is true?

13. In this lesson you discovered the Angle Bisector Conjecture. Write the converse of the Angle Bisector Conjecture. Do you think it's true? Why or why not?

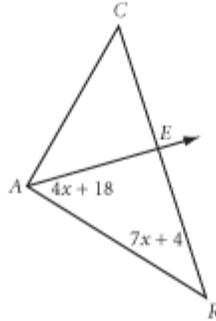
Notice how this mosaic floor at Church of Pomposa in Italy (ca. 850 c.e.) uses many duplicated shapes. What constructions do you see in the square pattern? Are all the triangles in the isosceles triangle pattern identical? How can you tell?



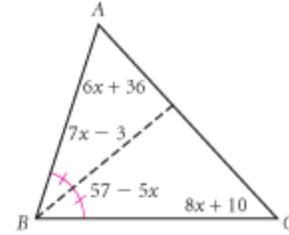
14. Solve for
- y
- .



15. If
- \overline{AE}
- bisects
- $\angle CAR$
- and
- $m\angle CAR = 84^\circ$
- , find
- $m\angle R$
- .



16. Which angle is largest,
- $\angle A$
- ,
- $\angle B$
- , or
- $\angle C$
- ?



Review

Draw or construct each figure in Exercises 17–21. Label the vertices with the appropriate letters. If you're unclear on the difference between "draw" and "construct," refer back to pages 144 and 145.

17. Draw a regular octagon. What traffic sign comes to mind?
18. Construct regular octagon *ALTOSIGN*.
19. Draw $\triangle ABC$ so that $AC = 3.5$ cm, $AB = 5.6$ cm, and $m\angle BAC = 130^\circ$.
20. Draw isosceles right $\triangle ABC$ so that $BC = 6.5$ cm and $m\angle B = 90^\circ$.
21. Draw a triangle with a 40° angle, a 60° angle, and a side between the given angles measuring 8 cm. Draw a second triangle with a 40° angle and a 60° angle but with a side measuring 8 cm *opposite* the 60° angle. Are the triangles congruent?
22. **Technology** Use geometry software to construct \overline{AB} and \overline{CD} , with point C on \overline{AB} and point D not on \overline{AB} . Construct the perpendicular bisector of \overline{CD} .
- Trace this perpendicular bisector as you drag point C along \overline{AB} . Describe the shape formed by this locus of lines.
 - Erase the tracings from part a. Now trace the midpoint of \overline{CD} as you drag C . Describe the locus of points.

IMPROVING YOUR VISUAL THINKING SKILLS

Coin Swap III

Arrange four dimes and four pennies in a row of nine squares, as shown. Switch the position of the four dimes and four pennies in exactly 24 moves. A coin can slide into an empty square next to it or can jump over one coin into an empty space. Record your solution by listing, in order, which type of coin is moved. For example, your list might begin PDPDPPDD

