LESSON 3.3

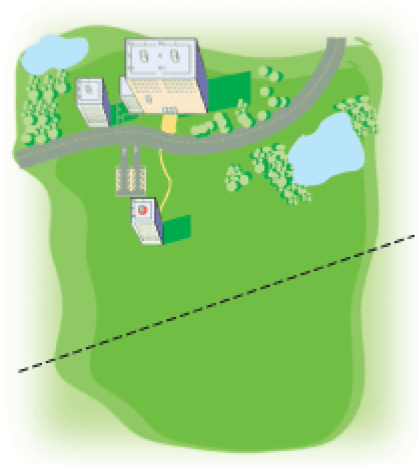
Constructing Perpendiculars to a Line

Intelligence plus character—that is the goal of true education.

MARTIN LUTHER KING, JR.

If you are in a room, look over at one of the walls. What is the distance from where you are to that wall? How would you measure that distance? There are a lot of distances from where you are to the wall, but in geometry when we speak of a distance from a point to a line we mean the perpendicular distance.

The construction of a perpendicular from a point to a line (with the point not on the line) is another of Euclid's constructions, and it has practical applications in many fields, including agriculture and engineering. For example, think of a high-speed Internet cable as a line and a building as a point not on the line. Suppose you wanted to connect the building to the Internet cable using the shortest possible length of connecting wire. How can you find out how much wire you need, so you don't buy too much?

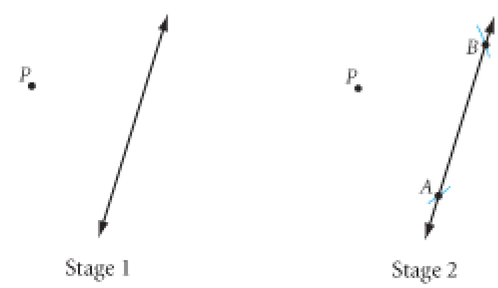


Investigation 1
Finding the Right Line

You will need

- a compass
- a straightedge

You already know how to construct perpendicular bisectors of segments. You can use that knowledge to construct a perpendicular from a point to a line.



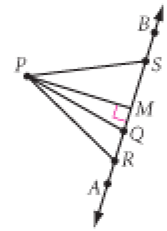
Step 1 Draw a line and a point labeled P not on the line, as shown above.

Step 2 Describe the construction steps you take at Stage 2.

Step 3 How is PA related to PB? What does this answer tell you about where point P lies? Hint: See the Converse of the Perpendicular Bisector Conjecture.

Step 4 Construct the perpendicular bisector of $\overline{AB}$. Label the midpoint M.

You have now constructed a perpendicular through a point not on the line. This is useful for finding the distance to a line.



Step 5 Label three randomly placed points on $\overleftrightarrow{AB}$ as Q, R, and S. Measure PQ, PR, PS, and PM. Which distance is shortest? Compare results with those of others in your group.

You are now ready to state your observations by completing the conjecture.

Shortest Distance Conjecture C-7

The shortest distance from a point to a line is measured along the __?__ from the point to the line.

Let's take another look. How could you use patty paper to do this construction?

Investigation 2

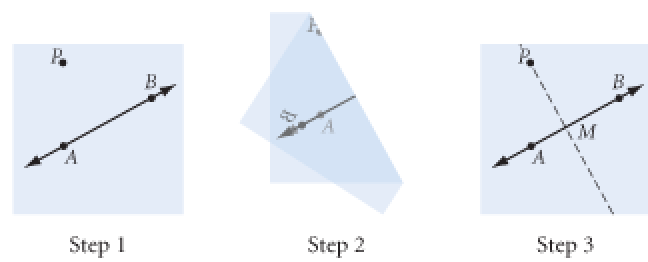
Patty-Paper Perpendiculars

You will need

- patty paper
- a straightedge

In Investigation 1, you constructed a perpendicular from a point to a line. Now let's do the same construction using patty paper.

On a piece of patty paper, perform the steps below.



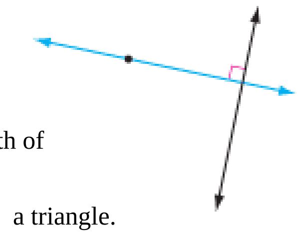
Step 1 Draw and label $\overleftrightarrow{AB}$ and a point P not on $\overleftrightarrow{AB}$.

Step 2 Fold the line onto itself, and slide the layers of paper so that point P appears to be on the crease. Is the crease perpendicular to the line? Check it with the corner of a piece of patty paper.

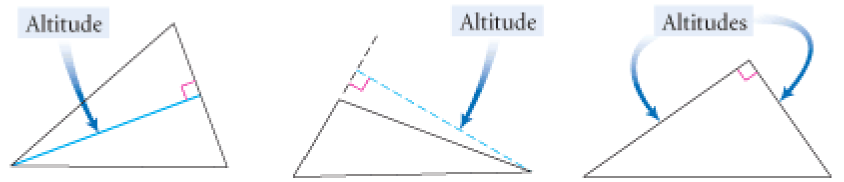
Step 3 Label the point of intersection M. Are $\angle AMP$ and $\angle BMP$ congruent? Supplementary? Why or why not?

In Investigation 2, is M the midpoint of $\overline{AB}$? Do you think it needs to be? Think about the techniques used in the two investigations. How do the techniques differ?

The construction of a perpendicular from a point to a line lets you find the shortest distance from a point to a line. The geometry definition of distance from a point to a line is based on this construction, and it reads, "The **distance from a point to a line** is the length of the perpendicular segment from the point to the line."



You can also use this construction to find an altitude of a triangle. An **altitude** of a triangle is a perpendicular segment from a vertex to the opposite side or to a line containing the opposite side.



An altitude can be inside the triangle. An altitude can be outside the triangle. An altitude can be one of the sides of the triangle.

The length of the altitude is the height of the triangle. A triangle has three different altitudes, so it has three different heights.

Exercises

Practice Your Skills

You will need

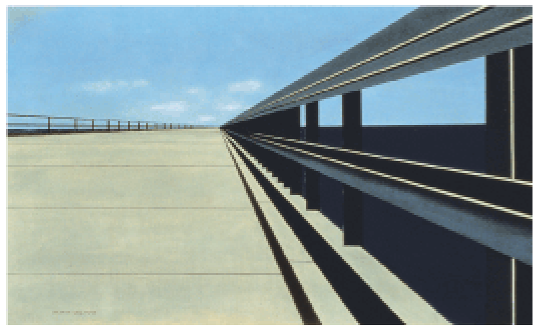
Construction tools for Exercises 1–12

Construction Use your compass and straightedge and the definition of distance to do Exercises 1–5.

1. Draw an obtuse angle *BIG*. Place a point *P* inside the angle. Now construct perpendiculars from the point to both sides of the angle. Which side is closer to point *P*?

2. Draw an acute triangle. Label it *ABC*. Construct altitude $\overline{CD}$ with point *D* on $\overleftrightarrow{AB}$. (We didn't forget about point *D*. It's at the *foot* of the perpendicular. Your job is to locate it.)

3. Draw obtuse triangle *OBT* with obtuse angle *O*. Construct altitude $\overline{BU}$. In an obtuse triangle, an altitude can fall outside the triangle. To construct an altitude from point *B* of your triangle, extend side $\overline{OT}$. In an obtuse triangle, how many altitudes fall outside the triangle and how many fall inside the triangle? (h)



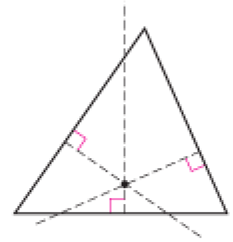
In this futuristic painting, American artist Ralston Crawford (1906–1978) has constructed a set of converging lines and vertical lines to produce an illusion of distance.

4. How can you construct a perpendicular to a line through a point that is on the line? Draw a line. Mark a point on your line. Now experiment. Devise a method to construct a perpendicular to your line at the point. (h)

5. Draw a line. Mark two points on the line and label them *Q* and *R*. Now construct a square *SQRE* with $\overline{QR}$ as a side. (h)

Construction For Exercises 6–9, use patty paper and a straightedge. (Attach your patty-paper work to your problems.)

6. Draw a line across your patty paper with a straightedge. Place a point *P* not on the line, and fold the perpendicular to the line through the point *P*. How would you fold to construct a perpendicular through a point on a line? Place a point *Q* on the line. Fold a perpendicular to the line through point *Q*. What do you notice about the two folds?



7. Draw a very large acute triangle on your patty paper. Place a point inside the triangle. Now construct perpendiculars from the point to all three sides of the triangle by folding. Mark your figure. How can you use your construction to decide which side of the triangle your point is closest to?

8. Construct an isosceles right triangle. Label its vertices *A*, *B*, and *C*, with point *C* the right angle. Fold to construct the altitude $\overline{CD}$. What do you notice about this line?

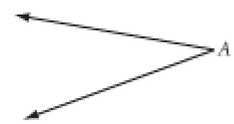
9. Draw obtuse triangle *OBT* with angle *O* obtuse. Fold to construct the altitude $\overline{BU}$. (Don't forget, you must extend the side $\overline{OT}$.)

Construction For Exercises 10–12, you may use either patty paper or a compass and a straightedge.

10. Construct a square *ABLE* given $\overline{AL}$ as a diagonal.

11. Construct a rectangle whose width is half its length.

12. Construct the complement of ∠*A*.

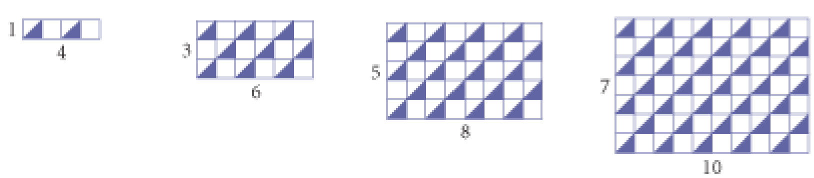


Review

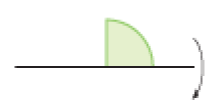
13. Copy and complete the table. Make a conjecture for the value of the *n*th term and for the value of the 35th term.

Rectangular pattern with triangles

Rectangle	1	2	3	4	5	6	...	*n*	...	35
Number of shaded triangles	2	9					...		...	



14. Sketch the solid of revolution formed when the two-dimensional figure at right is revolved about the line.



For Exercises 15–20, label the vertices with the appropriate letters. When you sketch or draw, use the special marks that indicate right angles, parallel segments, and congruent segments and angles.

15. Sketch obtuse triangle *FIT* with $m\angle I > 90°$ and median $\overline{IY}$.

16. Sketch $\overline{AB} \perp \overline{CD}$ and $\overline{EF} \perp \overline{CD}$.

17. Use your protractor to *draw* a regular pentagon. Draw all the diagonals. Use your compass to *construct* a regular hexagon. Draw three diagonals connecting alternating vertices. Do the same for the other three vertices. (h)

18. Draw a triangle with a 6 cm side and an 8 cm side and the angle between them measuring 40°. Draw a second triangle with a 6 cm side and an 8 cm side and exactly one 40° angle that is not between the two given sides. Are the two triangles congruent?

19. Sketch and label a polygon that has exactly three sides of equal length and exactly two angles of equal measure.

20. Sketch two triangles. Each should have one side measuring 5 cm and one side measuring 9 cm, but they should not be congruent.

project

CONSTRUCTING A TILE DESIGN

This Islamic design is based on two intersecting squares that form an 8-pointed star. Many designs of this kind can be constructed using only patty paper or a compass and a straightedge. Try it. Use construction tools to re-create this design or to create a design of your own based on an 8-pointed star.

Your project should include

- Your design based on an 8-pointed star, in color.
- A diagram showing your construction technique, with a written explanation of how you created it.

Here is a diagram to get you started.