Student Web Links @ Keymath.com

LESSON 3.2

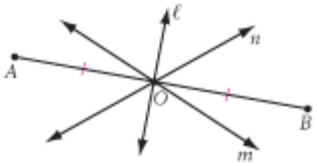
Constructing Perpendicular Bisectors

To be successful, the first thing to do is to fall in love with your work.
SISTER MARY LAURETTA

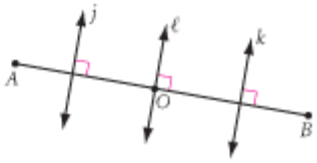
Each segment has exactly one midpoint. A **segment bisector** is a line, ray, or segment that passes through the midpoint of a segment.

A segment has many perpendiculars and many bisectors, but in a plane each segment has only one bisector that is also perpendicular to the segment. This line is its **perpendicular bisector.**

The construction of the perpendicular bisector of a segment creates a line of symmetry. You use this property when you hang a picture frame. If you want to center a picture above your desk, you need to place a nail in the wall somewhere along the perpendicular bisector of the segment that forms the top edge of your desk closest to the wall.



Lines ℓ, m, and n bisect $\overline{AB}$.



Lines j, k, and ℓ are perpendicular to $\overline{AB}$.

Line ℓ is the perpendicular bisector of $\overline{AB}$.

Investigation 1
Finding the Right Bisector

You will need
- patty paper
- a straightedge

In this investigation you will discover how to construct the perpendicular bisector of a segment.

Step 1

Step 2

Step 3

Step 1 Draw a segment on patty paper. Label it $\overline{PQ}$.

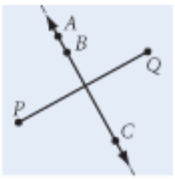
Step 2 Fold your patty paper so that endpoints P and Q land exactly on top of each other, that is, they **coincide.** Crease your paper along the fold.

Step 3 Unfold your paper. Draw a line in the crease. What is the relationship of this line to $\overline{PQ}$? Check with others in your group. Use your ruler and protractor to verify your observations.

How would you describe the relationship of the points on the perpendicular bisector to the endpoints of the bisected segment? There's one more step in your investigation.

Step 4

Place three points on your perpendicular bisector. Label them *A*, *B*, and *C*. With your compass, compare the distances *PA* and *QA*. Compare the distances *PB* and *QB*. Compare the distances *PC* and *QC*. What do you notice about the two distances from each point on the perpendicular bisector to the endpoints of the segment? Compare your results with the results of others. Then copy and complete the conjecture.



Remember to add each conjecture to your conjecture list and draw a figure for it.

Perpendicular Bisector Conjecture C-5

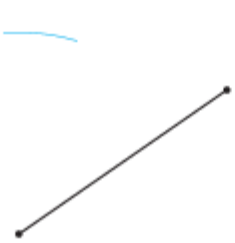
If a point is on the perpendicular bisector of a segment, then it is ?__ from the endpoints.

You've just completed the Perpendicular Bisector Conjecture. What about the converse of this statement?

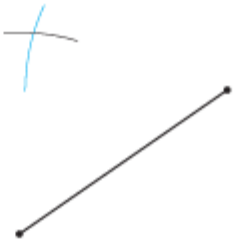
Investigation 2
Constructing the Perpendicular Bisector

You will need
- a compass
- a straightedge

If a point is **equidistant,** or the same distance, from two endpoints of a line segment in a plane, will it be on the segment's perpendicular bisector? If so, then locating two such points can help you construct the perpendicular bisector.



Step 1



Step 2

Step 1 Draw a line segment. Set your compass to more than half the distance between the endpoints. Using one endpoint as center, swing an arc on one side of the segment.

Step 2 Using the same compass setting, but using the other endpoint as center, swing a second arc intersecting the first.

Step 3 The point where the two arcs intersect is equidistant from the endpoints of your segment. Just as you did on one side of the segment, use your compass to find another such point. Use these points to construct a line. Is this line the perpendicular bisector of the segment? Use the paper-folding technique of Investigation 1 to check.

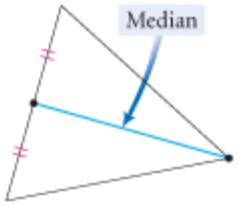
Step 4 Complete the conjecture below, and write a summary of what you did in this investigation.

Converse of the Perpendicular Bisector Conjecture C-6

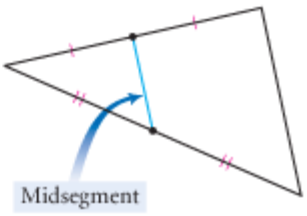
If a point is equidistant from the endpoints of a segment, then it is on the ? of the segment.

Notice that constructing the perpendicular bisector also locates the midpoint of a segment. Now that you know how to construct the perpendicular bisector and the midpoint, you can construct rectangles, squares, and right triangles. You can also construct two special segments in any triangle: medians and midsegments.

The segment connecting the vertex of a triangle to the midpoint of its opposite side is a **median.** There are three midpoints and three vertices in every triangle, so every triangle has three medians.



The segment that connects the midpoints of two sides of a triangle is a **midsegment.** A triangle has three sides, each with its own midpoint, so there are three midsegments in every triangle.



EXERCISES

Keymath.com Practice Your Skills

You will need

Construction tools for Exercises 1–10 and 14

Geometry software for Exercise 13

Construction For Exercises 1–5, construct the figures using only a compass and a straightedge.

1. Draw and label $\overline{AB}$. Construct the perpendicular bisector of $\overline{AB}$.
2. Draw and label $\overline{QD}$. Construct perpendicular bisectors to divide $\overline{QD}$ into four congruent segments. (h)
3. Draw a line segment so close to the edge of your paper that you can swing arcs on only one side of the segment. Then construct the perpendicular bisector of the segment. (h)
4. Using $\overline{AB}$ and $\overline{CD}$, construct a segment with length $2AB - \frac{1}{2}CD$. (h)

5. Construct $\overline{MN}$ with length equal to the average length of $\overline{AB}$ and $\overline{CD}$ above. (h)

6. ***Construction*** Do Exercises 1–5 using patty paper.

Construction For Exercises 7–10, you have your choice of construction tools. Use either a compass and a straightedge, or patty paper and a straightedge. Do *not* use patty paper and compass together.

7. Construct $\triangle ALI$. Construct the perpendicular bisector of each side. What do you notice about the three bisectors?

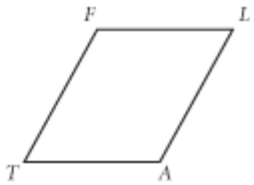
8. Construct $\triangle ABC$. Construct medians $\overline{AM}$, $\overline{BN}$, and $\overline{CL}$. Notice anything special?

9. Construct $\triangle DEF$. Construct midsegment $\overline{GH}$ where G is the midpoint of $\overline{DF}$ and H is the midpoint of $\overline{DE}$. What do you notice about the relationship between $\overline{EF}$ and $\overline{GH}$?

10. Copy rectangle *DSOE* onto your paper. Construct the midpoint of each side. Label the midpoint of $\overline{DS}$ point *I*, the midpoint of $\overline{SO}$ point *C*, the midpoint of $\overline{OE}$ point *V*, and the midpoint of $\overline{ED}$ point *R*. Construct quadrilateral *RICV*. Describe *RICV*.

11. The island shown at right has two post offices. The postal service wants to divide the island into two zones so that anyone within each zone is always closer to their own post office than to the other one. Copy the island and the locations of the post offices and locate the dividing line between the two zones. Explain how you know this dividing line solves the problem. Or pick several points in each zone and make sure they are closer to that zone's post office than they are to the other one.

12. Copy parallelogram *FLAT* onto your paper. Construct the perpendicular bisector of each side. What do you notice about the quadrilateral formed by the four lines?



13. ***Technology*** Use geometry software to construct a triangle. Construct a median. Are the two triangles created by the median congruent? Use an area measuring tool in your software program to find the areas of the two triangles. How do they compare? If you made the original triangle from heavy cardboard, and you wanted to balance that cardboard triangle on the edge of a ruler, what would you do?

14. ***Construction*** Construct a very large triangle on a piece of cardboard or mat board and construct its median. Cut out the triangle and see if you can balance it on the edge of a ruler. Sketch how you placed the triangle on the ruler. Cut the triangle into two pieces along the median and weigh the two pieces. Are they the same weight?

Review

In Exercises 15–20, match the term with its figure below.

15. Scalene acute triangle

16. Isosceles obtuse triangle

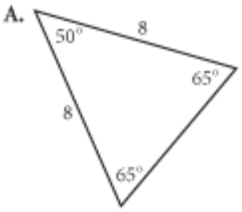
17. Isosceles right triangle

18. Isosceles acute triangle

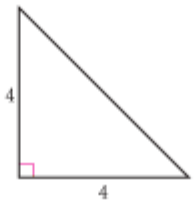
19. Scalene obtuse triangle

20. Scalene right triangle

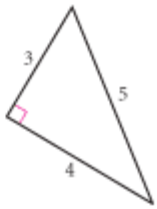
A.



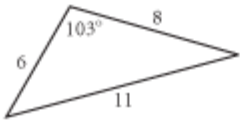
B.



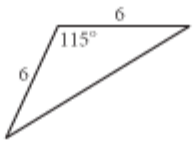
C.



D.



E.



F.

21. List the letters from the alphabet below that have a horizontal line of symmetry.

A B C D E F G H I J K L M N O P Q R S T U V W X Y Z

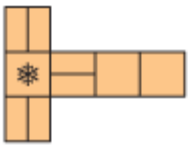
22. Use your ruler and protractor to draw a triangle with angle measures 40° and 70° and a side opposite the 70° angle with length 10 cm. Explain your method. Can you draw a second triangle using the same instructions that is not congruent to the first?

IMPROVING YOUR VISUAL THINKING SKILLS

Folding Cubes I

In the problems below, the figure at the left represents the net for a cube. When the net is folded, which cube at the right will it become?

1.

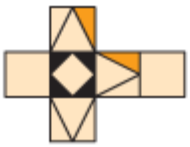


A.

B.

C.

2.

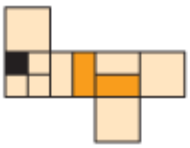


A.

B.

C.

3.



A.

B.

C.