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LESSON 2.4

Deductive Reasoning

That's the way things come clear. All of a sudden. And then you realize how obvious they've been all along.

MADELEINE L'ENGLE

Have you ever noticed that the days are longer in the summer? Or that mosquitoes appear after a summer rain? Over the years you have made conjectures, using inductive reasoning, based on patterns you have observed. When you make a conjecture, the process of discovery may not always help explain *why* the conjecture works. You need another kind of reasoning to help answer this question.

Deductive reasoning is the process of showing that certain statements follow logically from agreed-upon assumptions and proven facts. When you use deductive reasoning, you try to reason in an orderly way to convince yourself or someone else that your conclusion is valid. If your initial statements are true and you give a logical argument, then you have shown that your conclusion is true. For example, in a trial, lawyers use deductive arguments to show how the evidence that they present proves their case. A lawyer might make a very good argument. But first, the court must believe the evidence and accept it as true.

The success of an attorney's case depends on the jury accepting the evidence as true and following the steps in her deductive reasoning.

You use deductive reasoning in algebra. When you provide a reason for each step in the process of solving an equation, you are using deductive reasoning. Here is an example.

EXAMPLE A

Solve the equation for x. Give a reason for each step in the process.

$$3(2x + 1) + 2(2x + 1) + 7 = 42 - 5x$$

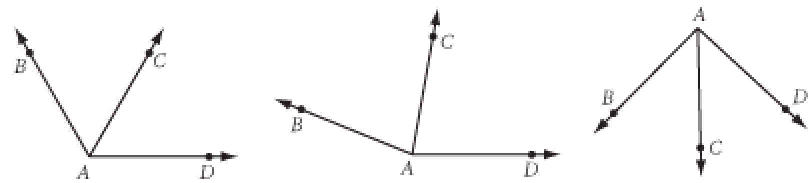
Solution

$3(2x + 1) + 2(2x + 1) + 7 = 42 - 5x$	The original equation.
$6x + 3 + 4x + 2 + 7 = 42 - 5x$	Distribute.
$10x + 12 = 42 - 5x$	Combine like terms.
$10x = 30 - 5x$	Subtract 12 from both sides.
$15x = 30$	Add $5x$ to both sides.
$x = 2$	Divide both sides by 15.

The next example shows how to use both kinds of reasoning: inductive reasoning to discover the property and deductive reasoning to explain why it works.

EXAMPLE B

In each diagram, $\overrightarrow{AC}$ bisects obtuse angle *BAD*. Classify $\angle BAD$, $\angle DAC$, and $\angle CAB$ as acute, right, or obtuse. Then complete the conjecture.



$m\angle BAD = 120°$ $m\angle BAD = 158°$ $m\angle BAD = 92°$

Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are _?_.

Justify your conjecture with a deductive argument.

Solution

In each diagram, $\angle BAD$ is obtuse because $m\angle BAD$ is greater than 90°. In each diagram, the angles formed by the bisector are acute because their measures—60°, 79°, and 46°—are less than 90°. So one possible conjecture is

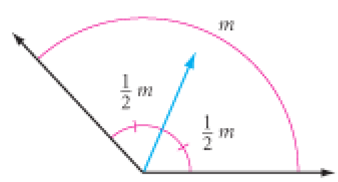
Conjecture: If an obtuse angle is bisected, then the two newly formed congruent angles are acute.

To explain why this is true, a useful reasoning strategy is to represent the situation algebraically. Let's use *m* to represent any angle measure.

Deductive Argument

By our definition, an angle measure is less than 180°.

$$m < 180°$$



When you bisect an angle, the newly formed angles each measure half the original angle.

$$\frac{1}{2}m < \frac{1}{2}(180°)$$

$$\frac{1}{2}m < 90°$$

The new angles measure less than 90°, so they are acute. ■

Inductive reasoning allows you to discover new ideas based on observed patterns. Deductive reasoning can help explain why your conjectures are true.

Good use of deductive reasoning depends on the quality of the argument. Just like the saying "A chain is only as strong as its weakest link," a deductive argument is only as good (or as true) as the statements used in the argument. A conclusion in a deductive argument is true only if *all* the statements in the argument are true and the statements in your argument clearly follow from each other.

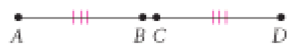
Inductive and deductive reasoning work very well together. In this investigation you will use inductive reasoning to form a conjecture. Then in your groups, you will use deductive reasoning to explain why it's true.

Investigation
Overlapping Segments

In each segment, $\overline{AB} \cong \overline{CD}$.

Step 1 From the markings on each diagram, determine the lengths of $\overline{AC}$ and $\overline{BD}$. What do you discover about these segments?

Step 2 Draw a new segment. Label it $\overline{AD}$. Place your own points B and C on $\overline{AD}$ so that $\overline{AB} \cong \overline{CD}$.



Step 3 Measure $\overline{AC}$ and $\overline{BD}$. How do these lengths compare?

Step 4 Complete the conclusion of this conjecture:

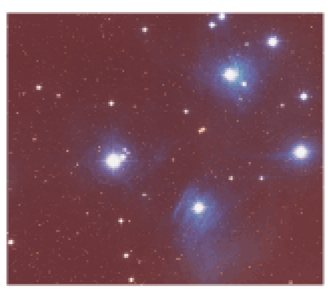
If $\overline{AD}$ has points A, B, C, and D in that order with $\overline{AB} \cong \overline{CD}$, then $\underline{\ ?\ }$.
(Overlapping Segments Conjecture)

Developing Proof In your groups, discuss how you can use logical reasoning to show that your conjecture from Step 4 will always be true. Remember, a useful reasoning strategy is to represent the situation algebraically. Then write down your ideas as a deductive argument. ■

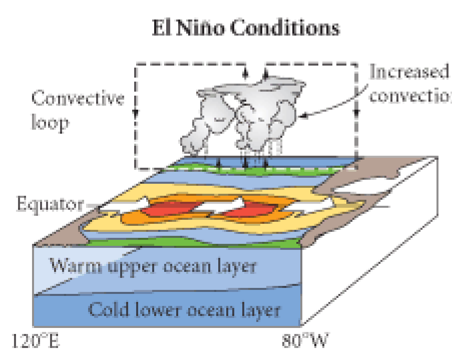
When you see this icon you will work with your group to develop a deductive argument or proof.

In the investigation you used inductive reasoning to discover the Overlapping Segments Conjecture. In your group discussion you then used deductive reasoning to explain why this conjecture is always true. You will use a similar process to discover and prove the Overlapping Angles Conjecture in Exercises 10 and 11.

Science CONNECTION



Here is an example of inductive reasoning, supported by deductive reasoning. El Niño is the warming of water in the tropical Pacific Ocean, which produces unusual weather conditions and storms worldwide. For centuries, farmers living in the Andes Mountains of South America observed that if the stars in the Pleiades constellation look dim in June, an El Niño year was coming. What is the connection? Scientists recently found that in an El Niño year, increased evaporation from the ocean produces high-altitude clouds that are invisible to the eye but create a haze that makes stars more difficult to see. The pattern that Andean farmers knew about for centuries is now supported by a scientific explanation. To find out more about this story, go to www.keymath.com/DG.

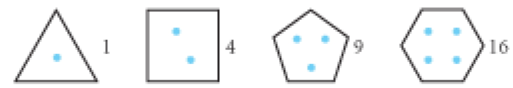


EXERCISES

1. When you use ___?___ reasoning, you are generalizing (making a conjecture) from careful observation that something is probably true. When you use ___?___ reasoning, you are establishing that if a set of properties is accepted as true, something else must be true.

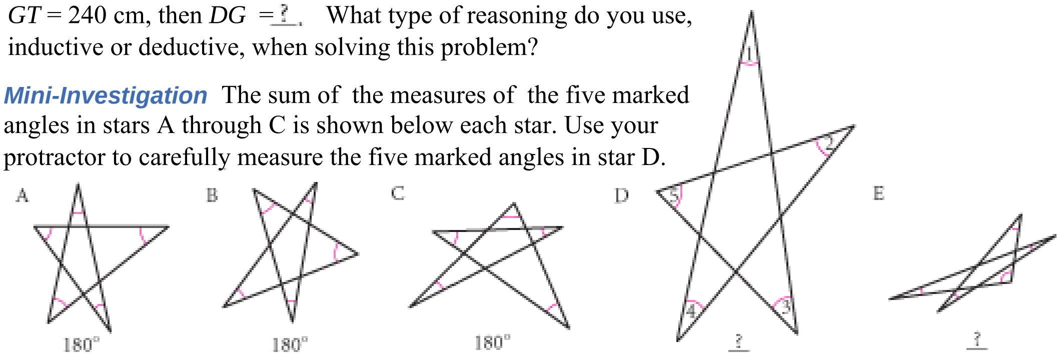
2. $\angle A$ and $\angle B$ are complementary. $m\angle A = 25°$. What is $m\angle B$? What type of reasoning do you use, inductive or deductive, when solving this problem?

3. If the pattern continues, what are the next two terms? What type of reasoning do you use, inductive or deductive, when solving this problem?



4. $\triangle DGT$ is isosceles with $TD = DG$. If the perimeter of $\triangle DGT$ is 756 cm and $GT = 240$ cm, then DG = ___?___. What type of reasoning do you use, inductive or deductive, when solving this problem?

5. ***Mini-Investigation*** The sum of the measures of the five marked angles in stars A through C is shown below each star. Use your protractor to carefully measure the five marked angles in star D.



If this pattern continues, without measuring, make a conjecture. What would be the sum of the measures of the marked angles in star E? What type of reasoning do you use, inductive or deductive, when solving this problem?

6. The definition of a parallelogram says, "If both pairs of opposite sides of a quadrilateral are parallel, then the quadrilateral is a parallelogram." Quadrilateral *LNDA* has both pairs of opposite sides parallel. What conclusion can you make? What type of reasoning did you use?

7. ***Developing Proof*** Using the ideas and algebra you discussed with your group, write a deductive argument for the Overlapping Segments Conjecture.

8. Use the Overlapping Segments Conjecture to complete each statement.

a. If $AB = 3$, then CD = ___?___.

b. If $AC = 10$, then BD = ___?___.

c. If $BC = 4$ and $CD = 3$, then AC = ___?___.

9. ***Developing Proof*** In Example B of this lesson you conjectured through inductive reasoning that if an obtuse angle is bisected, then the two newly formed congruent angles are acute. You then used deductive reasoning to explain why they were acute. Go back to the example and look at the sizes of the acute angles formed. What is the smallest possible size for the two congruent acute angles formed by the bisector of an obtuse angle? Use deductive reasoning to explain why. ⓗ

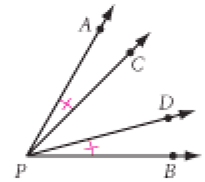
10. ***Mini-Investigation*** Do the geometry investigation and make a conjecture.

Given $\angle APB$ with points C and D in its interior and $m\angle APC = m\angle DPB$,

If $m\angle APD = 48°$, then $m\angle CPB = \underline{?}$

If $m\angle CPB = 17°$, then $m\angle APD = \underline{?}$

If $m\angle APD = 62°$, then $m\angle CPB = \underline{?}$



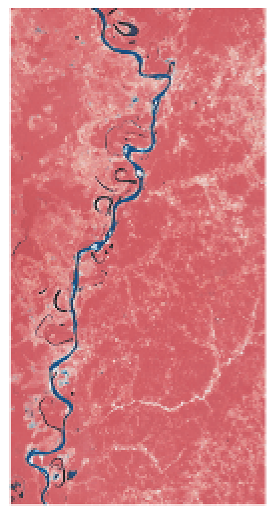
Conjecture: If points C and D lie in the interior of $\angle APB$, and $m\angle APC = m\angle DPB$, then $m\angle APD = \underline{?}$. (Overlapping Angles Conjecture)

11. ***Developing Proof*** Using reasoning similar to that in Exercise 7, write a deductive argument to explain why the Overlapping Angles Conjecture is true.

12. Think of a situation you observed outside of school in which deductive reasoning was used correctly. Write a paragraph or two describing what happened and explaining why you think it called for deductive reasoning.

Review

13. Mark Twain once observed that the lower Mississippi River is very crooked and that over the years, as the bends and the turns straighten out, the river gets shorter and shorter. Using numerical data about the length of the lower part of the river, he noticed that in the year 1700, the river was more than 1200 miles long, yet by the year 1875, it was only 973 miles long. Twain concluded that any person "can see that 742 years from now the lower Mississippi will be only a mile and three-quarters long." What is wrong with this inductive reasoning?



Aerial photo of the Mississippi River

For Exercises 14–16, use inductive reasoning to find the next two terms of the sequence.

14. 180, 360, 540, 720, $\underline{?}$, $\underline{?}$

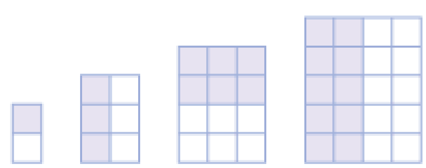
15. 0, 10, 21, 33, 46, 60, $\underline{?}$, $\underline{?}$

16. $\frac{1}{2}, 9, \frac{2}{3}, 10, \frac{3}{4}, 11, \underline{?}, \underline{?}$

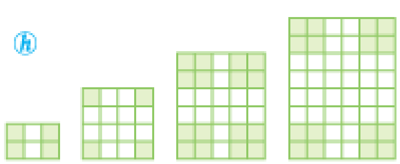
For Exercises 17–20, draw the next shape in each picture pattern.

17.

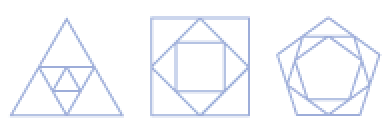
18.



19. ⓗ



20.



21. Think of a situation you have observed in which inductive reasoning was used incorrectly. Write a paragraph or two describing what happened and explaining why you think it was an incorrect use of inductive reasoning.

Match each term in Exercises 22–31 with one of the figures A–O.

22. Kite

23. Consecutive angles in a polygon

24. Trapezoid

25. Diagonal in a polygon

26. Pair of complementary angles

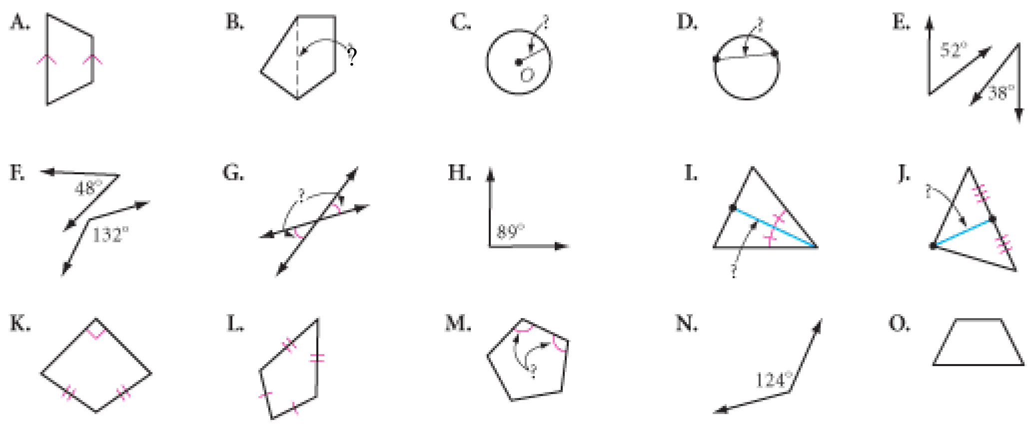
27. Radius

28. Pair of vertical angles

29. Chord

30. Acute angle

31. Angle bisector in a triangle



For Exercises 32–35, sketch and carefully label the figure.

32. Pentagon *WILDE* with $\angle ILD \cong \angle LDE$ and $\overline{LD} \cong \overline{DE}$

33. Isosceles obtuse triangle *OBG* with $m\angle BGO = 140°$

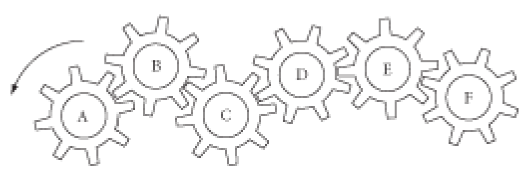
34. Circle *O* with a chord $\overline{CD}$ perpendicular to radius $\overline{OT}$

35. Circle *K* with acute angle *DKN* where *D* and *N* are points on circle *K*

IMPROVING YOUR VISUAL THINKING SKILLS

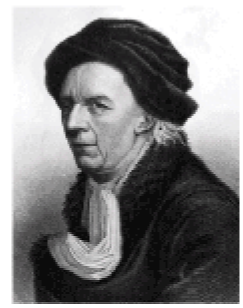
Rotating Gears

In what direction will gear E rotate if gear A rotates in a counterclockwise direction?

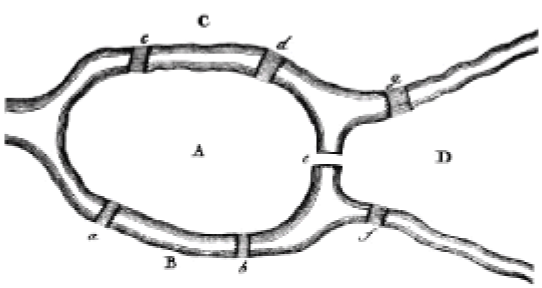


Exploration

The Seven Bridges of Königsberg


Leonhard Euler

The River Pregel (now Pregolya) runs through the university town of Königsberg (now Kaliningrad in Russia). In the middle of the river are two islands connected to each other and to the rest of the city by seven bridges. Many years ago, a tradition developed among the townspeople of Königsberg. They challenged one another to make a round trip over all seven bridges, walking over each bridge once and only once before returning to the starting point.



The seven bridges of Königsberg

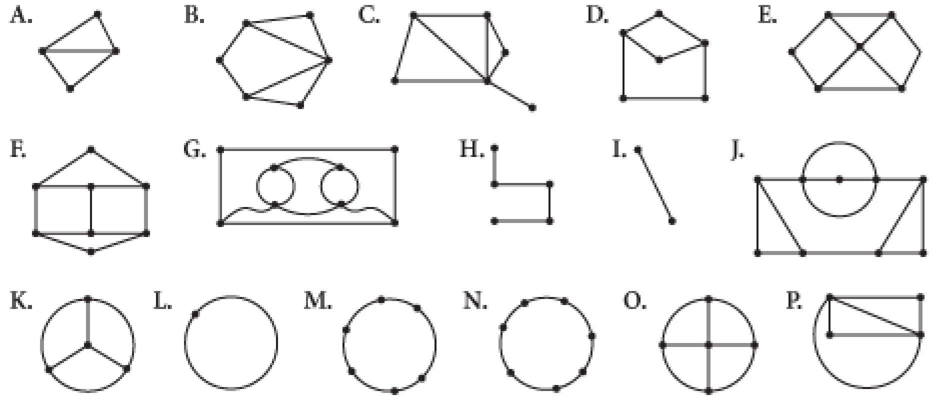
For a long time no one was able to do it, and yet no one was able to show that it couldn't be done. In 1735, they finally wrote to Leonhard Euler (1707–1783), a Swiss mathematician, asking for his help on the problem. Euler (pronounced "oyler") reduced the problem to a network of paths connecting the two sides of the rivers C and B, and the two islands A and D, as shown in the network above. Then Euler demonstrated that the task is impossible.

In this activity you will work with a variety of networks to see if you can come up with a rule to find out whether a network can or cannot be "traveled."

Activity
Traveling Networks

A collection of points connected by paths is called a **network.** When we say a network can be traveled, we mean that the network can be drawn with a pencil without lifting the pencil off the paper and without retracing any paths. (Points can be passed over more than once.)

Step 1 Try these networks and see which ones can be traveled and which are impossible to travel.

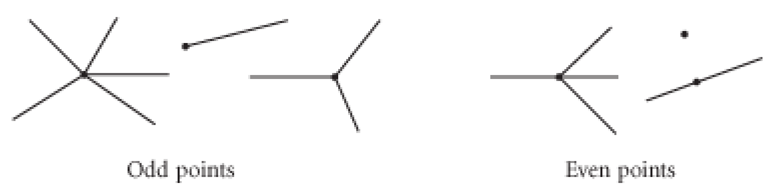


Which networks were impossible to travel? Are they impossible or just difficult? How can you be sure? As you do the next few steps, see whether you can find the reason why some networks are impossible to travel.

Step 2 Draw the River Pregel and the two islands shown on the first page of this exploration. Draw an eighth bridge so that you can travel over all the bridges exactly once if you start at point *C* and end at point *B*.

Step 3 Draw the River Pregel and the two islands. Can you draw an eighth bridge so that you can travel over all the bridges exactly once, starting and finishing at the same point? How many solutions can you find?

Step 4 Euler realized that it is the points of intersection that determine whether a network can be traveled. Each point of intersection is either "odd" or "even."



Did you find any networks that have only one odd point? Can you draw one? Try it. How about three odd points? Or five odd points? Can you create a network that has an odd number of odd points? Explain why or why not.

Step 5 How does the number of even points and odd points affect whether a network can be traveled?

Conjecture

A network can be traveled if ?.