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LESSON 2.2

# Finding the *n*th Term

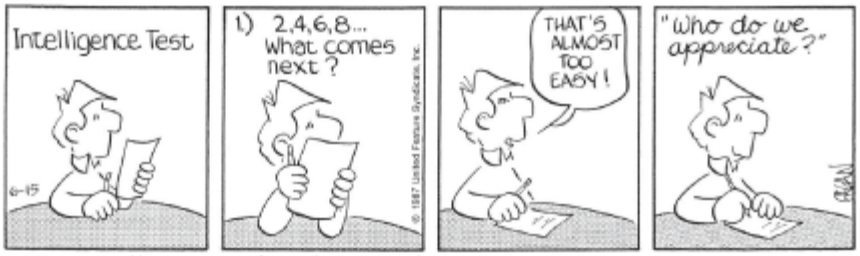
What would you do to get the next term in the sequence 20, 27, 34, 41, 48, 55, . . . ? A good strategy would be to find a pattern, using inductive reasoning. Then you would look at the differences between consecutive terms and predict what comes next. In this case there is a constant difference of +7. That is, you add 7 each time.

*If you do something once, people call it an accident. If you do it twice, they call it a coincidence. But do it a third time and you've just proven a natural law.*

GRACE MURRAY HOPPER

The next term is 55 + 7, or 62. What if you needed to know the value of the 200th term of the sequence? You certainly don't want to generate the next 193 terms just to get one answer. If you knew a rule for calculating *any* term in a sequence, without having to know the previous term, you could apply it to directly calculate the 200th term. The rule that gives the *n*th term for a sequence is called the **function rule.**

Let's see how the constant difference can help you find the function rule for some sequences.

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## Investigation
## Finding the Rule

Step 1

Copy and complete each table. Find the differences between consecutive values.

a.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $n - 5$ | −4 | −3 | −2 | | | | | |

b.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $4n - 3$ | 1 | 5 | 9 | | | | | |

c.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $-2n + 5$ | 3 | 1 | −1 | | | | | |

d.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $3n - 2$ | 1 | 4 | 7 | | | | | |

e.

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| $-5n + 7$ | 2 | −3 | −8 | | | | | |

Step 2 | Did you spot the pattern? If a sequence has constant difference 4, then then the number in front of the $n$ (the coefficient of $n$) is ?. In general, if the difference between the values of consecutive terms of a sequence is always the same, say $m$ (a constant), then the coefficient of n in the formula is ?.

Let's return to the sequence at the beginning of the lesson.

| Term | 1 | 2 | 3 | 4 | 5 | 6 | 7 | … | $n$ |
|---|---|---|---|---|---|---|---|---|---|
| Value | 20 | 27 | 34 | 41 | 48 | 55 | 62 | … | |

+7 +7

The constant difference is 7, so you know part of the rule is $7n$. How do you find the rest of the rule?

Step 3 | The first term ($n = 1$) of the sequence is 20, but if you apply the part of the rule you have so far, using $n = 1$, you get $7n = 7(1) = 7$, not 20. So how should you fix the rule? How can you get from 7 to 20? What is the rule for this sequence?

Step 4 | Check your rule by trying the rule with other terms in the sequence.

Let's look at an example of how to find a function rule for the $n$th term.

**EXAMPLE A** | Find the rule for the sequence 7, 2, −3, −8, −13, −18, . . .

▶ ***Solution*** | Placing the terms and values in a table we get

| Term | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ |
|---|---|---|---|---|---|---|---|---|
| Value | 7 | 2 | −3 | −8 | −13 | −18 | … | |

The difference between the terms is always −5. So the rule is

$-5n + something$

Let's use $c$ to stand for the unknown "something." So the rule is

$-5n + c$

To find $c$, replace the $n$ in the rule with a term number. Try $n = 1$ and set the expression equal to 7.

$-5(1) + c = 7$

$c = 12$

The rule is $-5n + 12$.

Rules for sequences can be expressed using **function notation.**

$f(n) = -5n + 12$

In this case, function $f$ takes an input value $n$, multiplies it by −5, and adds 12 to produce an output value.

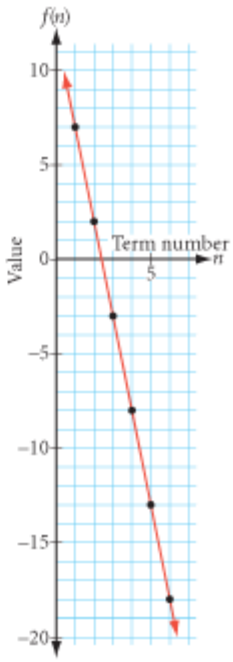
You can find the value of any term in the sequence by substituting the term number for $n$ into the function. To find the 20th term of this sequence, for instance, substitute 20 for $n$.

$f(20) = -5(20) + 12$

$f(20) = -88$

For functions in general, the input values for $n$ can be any number; but for sequences, the input values can only be positive integers.

Rules that generate a sequence with a constant difference are **linear functions.** To see why they're called linear, graph the term number and the value for the sequence as ordered pairs of the form (*term number, value*) on the coordinate plane. The values from Example A, shown in the table below, appear as points on the graph at right. Notice that the line $y = -5x + 12$ passes through these points.




| Term number $n$ | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ |
|---|---|---|---|---|---|---|---|---|
| Value $f(n)$ | 7 | 2 | −3 | −8 | −13 | −18 | … | $-5n + 12$ |

Let's look at an example of how to find the 200th term in a geometric pattern.

**EXAMPLE B**

If you place 200 points on a line, into how many non-overlapping rays and segments does it divide the line?

**Solution**

You need to find a rule that relates the number of points placed on a line to the number of parts created by those points. Then you can use your rule to answer the problem. Start by creating a table like the one shown below.

Sketch one point dividing a line. One point gives you just two rays. Enter that into the table.

Next, sketch two points dividing a line. This gives one segment and the two end rays. Enter these values into your table.

Next, sketch three points dividing a line, then four, then five, and so on. The table completed for one to three points is

| Points dividing the line | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| Non-overlapping rays | 2 | 2 | 2 | | | | … | | … | |
| Non-overlapping segments | 0 | 1 | 2 | | | | … | | … | |
| Total | 2 | 3 | 4 | | | | … | | … | |

Once you have values through 6 points, find the rule for each sequence. There are always two non-overlapping rays. The rule, or $n$th term, for the number of non-overlapping segments is $n - 1$. For 200 points there will be 199 segments. The $n$th term for the total number of distinct rays and segments is $n + 1$. For 200 points there will be 201 distinct parts of the line.

## EXERCISES

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For Exercises 1–3, find the function rule $f(n)$ for each sequence. Then find the 20th term in the sequence.

**1.** (h)

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(n)$ | 3 | 9 | 15 | 21 | 27 | 33 | … | | … | |

**2.**

| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(n)$ | 1 | −2 | −5 | −8 | −11 | −14 | … | | … | |

**3.**

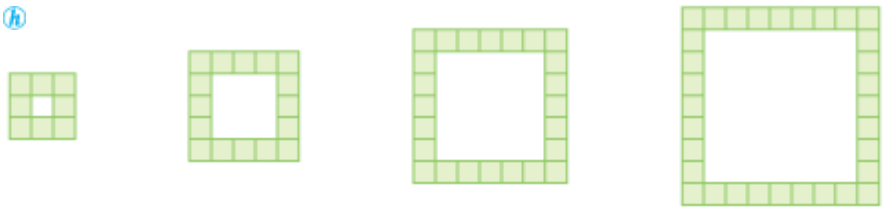
| $n$ | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 20 |
|---|---|---|---|---|---|---|---|---|---|---|
| $f(n)$ | −4 | 4 | 12 | 20 | 28 | 36 | … | | … | |

**4.** How many triangles are formed when you draw all the possible diagonals from just one vertex of a 35-gon? (h)

| Number of sides | 3 | 4 | 5 | 6 | … | $n$ | … | 35 |
|---|---|---|---|---|---|---|---|---|
| Number of triangles formed | | | | | … | | … | |

For Exercises 5–7, find the rule for the $n$th figure. Then find the number of colored tiles or matchsticks in the 200th figure.

**5.** (h)

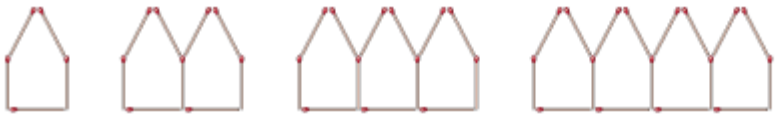


| Figure number | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of tiles | 8 | | | | | | … | | … | |

**6.**

| Figure number | 1 | 2 | 3 | 4 | 5 | 6 | … | $n$ | … | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of tiles | | 5 | | | | | … | | … | |

**7.**

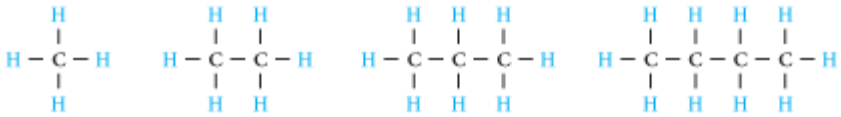


| Figure number | 1 | 2 | 3 | 4 | 5 | 6 | … | *n* | … | 200 |
|---|---|---|---|---|---|---|---|---|---|---|
| Number of matchsticks | 5 | 9 | | | | | … | | … | |
| Number of matchsticks in perimeter of figure | 5 | 8 | | | | | … | | … | |

**8.** Graph the values in your tables from Exercises 5–7. Which set of points lies on a steeper line? What number in the rule gives a measure of steepness?

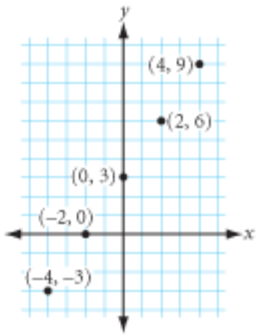
**9.** ***Application*** Hydrocarbons are molecules that consist of carbon (C) and hydrogen (H). Hydrocarbons in which all the bonds between the carbon atoms are single bonds are called *alkanes*. The first four alkanes are modeled below.

Sketch the alkane with eight carbons in the chain. What is the general rule for alkanes ($C_nH_?$) ? In other words, if there are $n$ carbon atoms (C), how many hydrogen atoms (H) are in the alkane?



Methane $(CH_4)$ Ethane $(C_2H_6)$ Propane $(C_3H_8)$ Butane $(C_4H_{10})$

**10.** Find the rule for the set of points in the graph shown at right. Place the $x$-coordinate of each ordered pair in the top row of your table and the corresponding $y$-coordinate in the second row. What is the value of $y$ in terms of $x$?




## Review

For Exercises 11–14, sketch and carefully label the figure.

**11.** Equilateral triangle $EQL$ with $\overline{QT}$ where $T$ lies on $\overline{EL}$ and $\overline{QT} \perp \overline{EL}$

**12.** Isosceles obtuse triangle $OLY$ with $\overline{OL} \cong \overline{YL}$ and angle bisector $\overline{LM}$

**13.** A cube with a plane passing through it; the cross section is rectangle $RECT$

**14.** A net for a rectangular solid with the dimensions 1 by 2 by 3 cm

**15.** Márisol's younger brother José was drawing triangles when he noticed that every triangle he drew turned out to have two sides congruent. José conjectured: "Look, Márisol, all triangles are isosceles." How should Márisol have responded?

**16.** Use your ruler and protractor to draw a triangle with side lengths 8 cm and 9 cm and an angle between them measuring 45°. Explain your method. Can you draw a second triangle using the same instructions that is not congruent to the first?

**17.** Tanya's favorite lunch is peanut butter and jelly on wheat bread with a glass of milk. Lately, she has been getting an allergic reaction after eating this lunch. She is wondering if she might be developing an allergy to peanut butter, wheat, or milk. What experiment could she do to find out which food it might be? What type of reasoning would she be using?

**18.** The sequence 3, 8, 15, 24, . . . is another rectangular number pattern. How many squares are there in the 50th rectangular array? (h)


## project

### BEST-FIT LINES

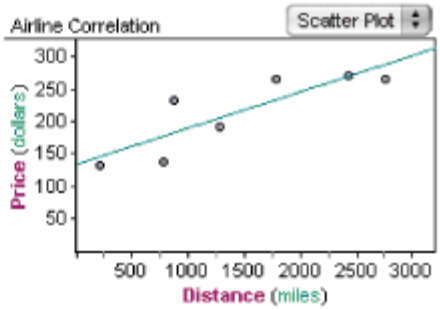
Fathom 2 Dynamic Data Software

With Fathom Dynamic Statistics™ software, you can plot your data points and find the linear equation that best fits your data.

The following table and graph show the mileage and lowest priced round-trip airfare between New York City and each destination city on March 11, 2006. Is there a relationship between the money you spend and how far you can travel?

| Destination City | Distance (miles) | Price ($) |
|---|---|---|
| Boston | 215 | $133 |
| Chicago | 784 | $139 |
| Atlanta | 865 | $232 |
| Miami | 1286 | $192 |
| Denver | 1791 | $267 |
| Phoenix | 2431 | $272 |
| Los Angeles | 2763 | $267 |

Source: http://www.Expedia.com




Even though the data are not linear, you can find a linear equation that *approximately* fits the data. The graph of this equation is called a **line of best fit.** How would you use a line of best fit to predict the cost of a round-trip ticket to Seattle (2814 miles)? How would you use it to determine how far you could travel (in miles) with $250? How accurate do you think the answer would be?

Choose a topic and a relationship to explore. You can use data from the census (such as age and income), or data you collect yourself (such as number of ice cubes in a glass and melting time). For more sources and ideas, go to www.keymath.com/DG.

Your project should include

- A collection of data points with the topic, relationship, and source clearly presented.
- An accurate graph showing a line of best fit.
- At least two predictions using your line, and a summary of your results including how accurate you think your predictions are.