LESSON 2.1

Inductive Reasoning

We have to reinvent the wheel every once in a while, not because we need a lot of wheels; but because we need a lot of inventors.

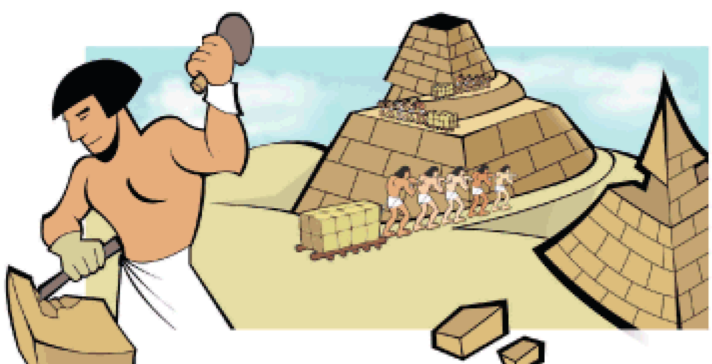
BRUCE JOYCE

As a child, you learned by experimenting with the natural world around you. You learned how to walk, to talk, and to ride your first bicycle, all by trial and error. From experience you learned to turn a water faucet on with a counterclockwise motion and to turn it off with a clockwise motion. You achieved most of your learning by a process called **inductive reasoning.** It is the process of observing data, recognizing patterns, and making generalizations about those patterns.

Geometry is rooted in inductive reasoning. In ancient Egypt and Babylonia, geometry began when people developed procedures for measurement after much experience and observation. Assessors and surveyors used these procedures to calculate land areas and to reestablish the boundaries of agricultural fields after floods. Engineers used the procedures to build canals, reservoirs, and the Great Pyramids. Throughout this course you will use inductive reasoning. You will perform investigations, observe similarities and patterns, and make many discoveries that you can use to solve problems.

Language CONNECTION

The word "geometry" means "measure of the earth" and was originally inspired by the ancient Egyptians. The ancient Egyptians devised a complex system of land surveying in order to reestablish land boundaries that were erased each spring by the annual flooding of the Nile River.



Inductive reasoning guides scientists, investors, and business managers. All of these professionals use past experience to assess what is likely to happen in the future.

When you use inductive reasoning to make a generalization, the generalization is called a **conjecture.** Consider the following example from science.

EXAMPLE A

A scientist dips a platinum wire into a solution containing salt (sodium chloride), passes the wire over a flame, and observes that it produces an orange-yellow flame.

She does this with many other solutions that contain salt, finding that they all produce an orange-yellow flame. Make a conjecture based on her findings.

Platinum wire flame test

▶ Solution

The scientist tested many other solutions containing salt and found no counterexamples. You should conjecture: "If a solution contains sodium chloride, then in a flame test it produces an orange-yellow flame."

Like scientists, mathematicians often use inductive reasoning to make discoveries. For example, a mathematician might use inductive reasoning to find patterns in a number sequence. Once he knows the pattern, he can find the next term.

EXAMPLE B

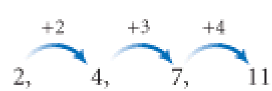
Consider the sequence

2, 4, 7, 11, . . .

Make a conjecture about the rule for generating the sequence. Then find the next three terms.

▶ Solution

Look at the numbers you add to get each term. The 1st term in the sequence is 2. You add 2 to find the 2nd term. Then you add 3 to find the 3rd term, and so on.

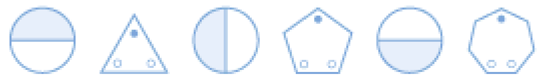


You can conjecture that if the pattern continues, you always add the next counting number to get the next term. The next three terms in the sequence will be 16, 22, and 29.

In the following investigation you will use inductive reasoning to recognize a pattern in a series of drawings and use it to find a term much farther out in a sequence.

Investigation
Shape Shifters

Look at the sequence of shapes below. Pay close attention to the patterns that occur in every other shape.



Step 1 What patterns do you notice in the 1st, 3rd, and 5th shapes?

Step 2 What patterns do you notice in the 2nd, 4th, and 6th shapes?

Step 3 Draw the next two shapes in the sequence.

Step 4 Use the patterns you discovered to draw the 25th shape.

Step 5 Describe the 30th shape in the sequence. You do not have to draw it!

Sometimes a conjecture is difficult to find because the data collected are unorganized or the observer is mistaking coincidence with cause and effect. Good use of inductive reasoning depends on the quantity and quality of data. Sometimes not enough information or data have been collected to make a proper conjecture. For example, if you are asked to find the next term in the pattern 3, 5, 7, you might conjecture that the next term is 9—the next odd number. Someone else might notice that the pattern is the consecutive odd primes and say that the next term is 11. If the pattern were 3, 5, 7, 11, 13, what would you be more likely to conjecture?

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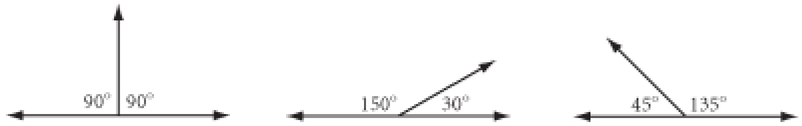
EXERCISES

1. On his way to the local Hunting and Gathering Convention, caveperson Stony Grok picks up a rock, drops it into a lake, and notices that it sinks. He picks up a second rock, drops it into the lake, and notices that it also sinks. He does this five more times. Each time, the rock sinks straight to the bottom of the lake. Stony conjectures: "Ura nok seblu," which translates to _?_. What counterexample would Stony Grok need to find to disprove, or at least to refine, his conjecture? (h)

2. Sean draws these geometric figures on paper. His sister Courtney measures each angle with a protractor. They add the measures of each pair of angles to form a conjecture. Write their conjecture.



For Exercises 3–10, use inductive reasoning to find the next two terms in each sequence.

3. 1, 10, 100, 1000, ?, ?

4. $\frac{1}{6}, \frac{1}{3}, \frac{1}{2}, \frac{2}{3}$, ?, ? (h)

5. 7, 3, −1, −5, −9, −13, ?, ?

6. 1, 3, 6, 10, 15, 21, ?, ?

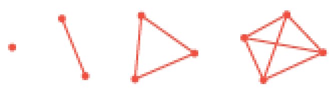
7. 1, 1, 2, 3, 5, 8, 13, ?, ? (h)

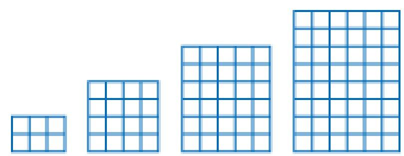
8. 1, 4, 9, 16, 25, 36, ?, ? (h)

9. 32, 30, 26, 20, 12, 2, ?, ?

10. 1, 2, 4, 8, 16, 32, ?, ?

For Exercises 11–16, use inductive reasoning to draw the next shape in each picture pattern.

11. 

12. 

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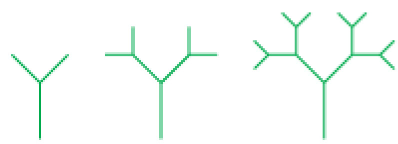
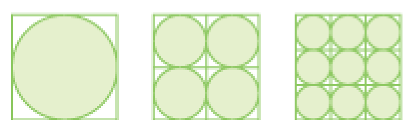
14. 

15. 

16. 

Use the rule provided to generate the first five terms of the sequence in Exercise 17 and the next five terms of the sequence in Exercise 18.

17. $3n - 2$ (h)

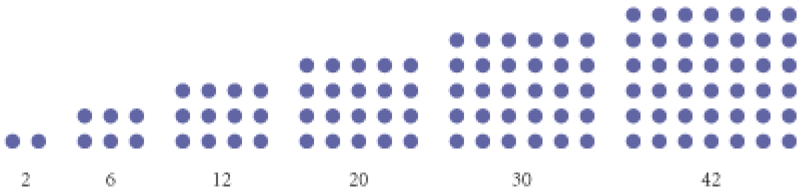
18. 1, 3, 6, 10, . . . $\frac{n(n+1)}{2}$, . . .

19. Now it's your turn. Generate the first five terms of a sequence. Give the sequence to a member of your family or to a friend and ask him or her to find the next two terms in the sequence. Can he or she find your pattern?

20. Write the first five terms of two different sequences in which 12 is the 3rd term.

21. Think of a situation in which you have used inductive reasoning. Write a paragraph describing what happened and explaining why you think you used inductive reasoning. (h)

22. The sequence 2, 6, 12, 20, 30, 42, . . . is called a rectangular number pattern because the terms can be visualized as rectangular arrangements of dots. What would be the 7th term in this sequence? What would be the 10th term? The 25th term ?



23. Look at the pattern in these pairs of equations. Decide if the conjecture is true. If it is not true, find a counterexample.

$12^2 = 144$ and $21^2 = 441$

$13^2 = 169$ and $31^2 = 961$

$103^2 = 10609$ and $301^2 = 90601$

$112^2 = 12544$ and $211^2 = 44521$

Conjecture: If two numbers have the same digits in reverse order, then the squares of those numbers will have identical digits, but in reverse order.

24. Study the pattern and make a conjecture by completing the fifth line. What would be the conjecture for the sixth line? What about for the tenth line?

$1 \cdot 1 = 1$

$11 \cdot 11 = 121$

$111 \cdot 111 = 12,321$

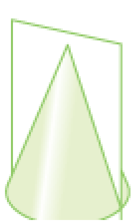
$1,111 \cdot 1,111 = 1,234,321$

$11,111 \cdot 11,111 = \underline{?}$

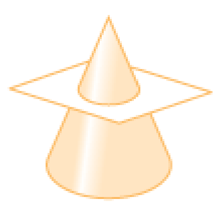
Review

For Exercises 25–27, sketch the section formed when the cone is sliced by the plane, as shown.

25.

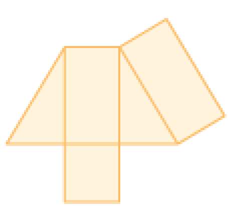


26.

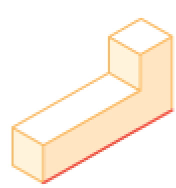


27.

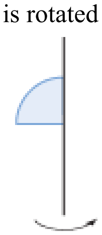
28. Sketch the three-dimensional figure formed by folding the net below into a solid.



29. Sketch the figure shown below, but with the red edge vertical and facing you.



30. Sketch the solid of revolution formed when the two-dimensional figure is rotated about the line.



For Exercises 31–40, write the word that makes the statement true.

31. Points are _?_ if they lie on the same line.

32. A triangle with two congruent sides is _?_.

33. A polygon with 12 sides is called a(n) _?_.

34. A trapezoid has exactly one pair of _?_ sides.

35. The geometry tool used to measure the size of an angle in degrees is called a(n) _?_.

36. A(n) _?_ of a circle connects its center to a point on the circle.

37. A segment connecting any two non-adjacent vertices in a polygon is called a(n) _?_.

38. A(n) _?_ polygon is both equiangular and equilateral.

39. If angles are complementary, then their measures add to _?_.

40. If two lines intersect to form a right angle, then they are _?_.

For Exercises 41–44, sketch and label the figure.

41. Pentagon *GIANT* with diagonal $\overline{AG}$ parallel to side $\overline{NT}$

42. A quadrilateral that has reflectional symmetry, but not rotational symmetry

43. A prism with a hexagonal base

44. A counterexample to show that the following statement is false: The diagonals of a kite bisect the angles. (h)

45. Use your ruler and protractor to draw a triangle with angles measuring 40° and 60° and a side between them with length 9 cm. Explain your method. Can you draw a second triangle using the same instructions that is not congruent to the first?

IMPROVING YOUR REASONING SKILLS

Puzzling Patterns

These patterns are "different." Your task is to find the next term.

1. 18, 46, 94, 63, 52, 61, _?_
2. O, T, T, F, F, S, S, E, N, _?_
3. 1, 4, 3, 16, 5, 36, 7, _?_
4. 4, 8, 61, 221, 244, 884, _?_
5. 6, 8, 5, 10, 3, 14, 1, _?_
6. B, 0, C, 2, D, 0, E, 3, F, 3, G, _?_
7. 2, 3, 6, 1, 8, 6, 8, 4, 8, 4, 8, 3, 2, 3, 2, 3, _?_
8. A E F H I K L M N T V W
   B C D G J O P Q R S U
   Where do the X, Y, and Z go?