2.2 Deductive Reasoning

Objective:

- **I CAN use inductive and deductive reasoning to make and defend conjectures.**

Two Types of Reasoning

Deductive

Start with lots of rules.

↓

Make another rule.

Inductive

Establish a rule.

↑

Start with lots of observations.

Two Types of Reasoning Con't

Deductive

Gravity makes things fall downwards.

Things that fall from a great height get hurt.

↓

If I jump off this building, I will fall downwards.

Inductive

Things I throw off the roof fall down.

↑

I threw a ball off the roof and it fell down.
I threw a rock off the roof and it fell down.
I threw a cat off the roof and it fell down.

Two Types of Reasoning Con't

Deductive	Inductive
All tall people are handsome.	All tall people are handsome.
Handsome people have lots of friends.	↑
↓	
All tall people have lots of friends.	Mr. X is tall and handsome. Tim Robbins is tall and handsome.

Inductive Reasoning	Deductive Reasoning
• Uses observations to make generalizations	• Uses a series of statements to prove a generalization true.
• If I burn my hand after 5 times of touching the stove, I can conclude that every time I touch I will burn my hand.	• If A, then B. • If 2x = 10, then x = 5.

Conditional Statement

Definition: A **conditional** statement is a statement that can be written in if-then form.
"***If*** _______________, ***then*** ________________."

Example 1: **If** your feet smell and your nose runs, **then** you're built upside down.

Continued……

Conditional Statement - *continued*

Conditional Statements have two parts:

The **hypothesis** is the part of a conditional statement that follows **"if"** (when written in if-then form.)

The hypothesis is the given information, or the condition.

The **conclusion** is the part of an if-then statement that follows **"then"** (when written in if-then form.)

The conclusion is the result of the given information.

Writing Conditional Statements

Conditional statements can be written in **"if-then"** form to emphasize which part is the hypothesis and which is the conclusion.

Hint: *Turn the subject into the hypothesis.*

Example 1: Vertical angles are congruent. ***can be written as...***

Conditional Statement: **If two angles are vertical, then they are congruent.**

Example 2: Seals swim. ***can be written as...***

Conditional Statement: **If an animal is a seal, then it swims.**

If …Then *vs.* Implies

Another way of writing an if-then statement is using the word implies.

If two angles are vertical, then they are congruent.

Two angles are vertical *implies* they are congruent.

Conditional Statements can be true or false:

- A conditional statement is false **only when** the hypothesis is true, but the conclusion is false.

- A **counterexample** is an example used to show that a statement is not always true and therefore false.

Statement: **If you live in Florida, then you live in Miami Shores.**

Is there a counterexample?

Yes !!!

Counterexample: **I live in Florida, BUT I live Orlando.**

Therefore (∴) the statement is false.

Symbolic Logic

- Symbols can be used to modify or connect statements.
- Symbols for Hypothesis and Conclusion:

Hypothesis is represented by "***p***".

Conclusion is represented by "***q***".

if p, then q
or
p implies q

Continued…..

Symbolic Logic - *continued*

$p \rightarrow q$ *is used to represent* if p, then q

or

p implies q

Example: p: a number is prime
q: a number has exactly two divisors

$p \rightarrow q$: If a number is prime, then it has exactly two divisors.

Continued.....

Symbolic Logic - *continued*

~ *is used to represent the word* "not"

Example 1: p: The angle is obtuse

~p: The angle is not obtuse

Note: ~p means that the angle could be acute, right, or straight.

Example 2: p: I am not happy

~p: I am happy

~p took the "not" out- it would have been a double negative (not not)

Symbolic Logic - *continued*

$\wedge$ *is used to represent the word* **"and"**

Example: p: a number is even

q: a number is divisible by 3

$p \wedge q$: A number is even and it is divisible by 3.

i.e. 6,12,18,24,30,36,42...

Symbolic Logic- *continued*

v *is used to represent the word* "or"

Example: p: a number is even

q: a number is divisible by 3

pvq: A number is even or it is divisible by 3.

i.e. 2,3,4,6,8,9,10,12,14,15,...

Symbolic Logic - continued

$\therefore$ *is used to represent the word* "therefore"

Example: Therefore, the statement is false.

$\therefore$ the statement is false

Forms of Conditional Statements

Converse: **Switch the hypothesis and conclusion ($q \rightarrow p$)**

$p \rightarrow q$ ***If*** **two angles are vertical**, ***then*** **they are congruent.**

$q \rightarrow p$ ***If*** **two angles are congruent**, ***then*** **they are vertical.**

Continued…..

Forms of Conditional Statements

Inverse: State the opposite of both the hypothesis and conclusion. $(\sim p \rightarrow \sim q)$

$p \rightarrow q$: ***If*** **two angles are vertical,** ***then*** **they are congruent.**

$\sim p \rightarrow \sim q$: ***If*** **two angles are <u>not</u> vertical,** ***then*** **they are <u>not</u> congruent.**

Forms of Conditional Statements

Contrapositive: Switch the hypothesis and conclusion and state their opposites. ($\sim q \rightarrow \sim p$)

$p \rightarrow q$: ***If*** **two angles are vertical,** ***then*** **they are congruent.**

$\sim q \rightarrow \sim p$: ***If*** **two angles are not congruent,** ***then*** **they are not vertical.**

Forms of Conditional Statements

- Contrapositives are logically equivalent to the original conditional statement.
- If $p \rightarrow q$ is true, then $\sim q \rightarrow \sim p$ is true.
- If $p \rightarrow q$ is false, then $\sim q \rightarrow \sim p$ is false.

Biconditional

- When a conditional statement and its converse are both true, the two statements may be combined.
- Use the phrase if and only if (sometimes abbreviated: iff)

Statement: If an angle is right then it has a measure of 90°.

Converse: If an angle measures 90°, then it is a right angle.

Biconditional: An angle is right if and only if it measures 90°.

Vocabulary

Conditional Statement:

hypothesis & conclusion

If-Then Form:

If hypothesis, then conclusion.

Ex: All math classes use numbers.

If the class is math,
then it uses numbers.

Negation: opposite (not)

Ex: Math isn't the best class ever.

Math is the best class ever.

Vocabulary

Converse:

If conclusion, then hypothesis.

Ex: If an angle is 90°, then it's right.

If an angle is right, then it's 90°.

Inverse:

If NOT hypothesis, then NOT conclusion.

Ex: If an angle is 90°, then it's right.

If an angle is not 90°, then it's not right.

Contrapositive:

If NOT conclusion, then NOT hypothesis.

Ex: If an angle is 90°, then it's right.

If an angle is not right, then it's not 90°.

Vocabulary

Perpendicular Lines:

If 2 lines intersect to form a right angle, then they are perpendicular lines.

If 2 lines are perpendicular, then they intersect to form a right angle.

2 lines are perpendicular if and only if they intersect to form a right angle.

Biconditional Statement:

statement and its converse are true "if and only if"

Vocabulary

Deductive Reasoning:

proof using definitions, postulates, theorems

Law of Detachment:

If the hypothesis is true, then the conclusion is true.

Law of Syllogism:

If p, then q.
If q, then r.
If p, then r.

Example #1
What conclusion can you draw based on the two conditional statements below?

$$If\ \ a = 4,\ then\ 4a = 16.$$

$$If\ \ 3a = 12,\ then\ a = 4.$$

Law of Syllogism

$$If\ \ 3a = 12,\ then\ a = 4.$$

$$If\ \ a = 4,\ then\ 4a = 16.$$

$$If\ \ 3a = 12,\ then\ 4a = 16.$$

Example #2

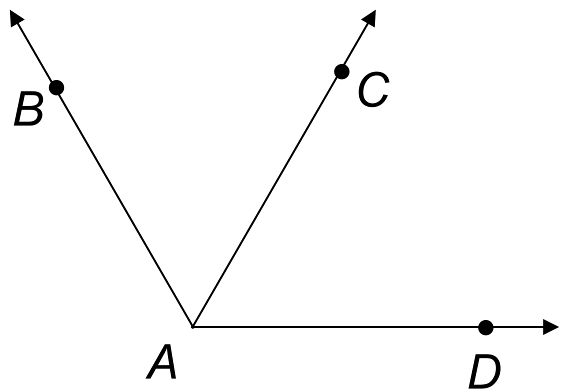
Solve the equation for *x*. Give a reason for each step in the process.

$$3(2x+1)+2(2x+1)+7=42-5x$$

$3(2x+1)+2(2x+1)+7=42-5x$	Original Equation
$5(2x+1)+7=42-5x$	Combine Like Terms
$10x+5+7=42-5x$	Distribute
$10x+12=42-5x$	Combine Like Terms
$10x=30-5x$	Subtract 12
$15x=30$	Add 5x
$x=2$	Divide by 15

Example #3

In each diagram ray AC bisects obtuse angle BAD.
Classify each angle CAD as acute, right, or obtuse.
Then write and prove a conjecture about the angles formed.

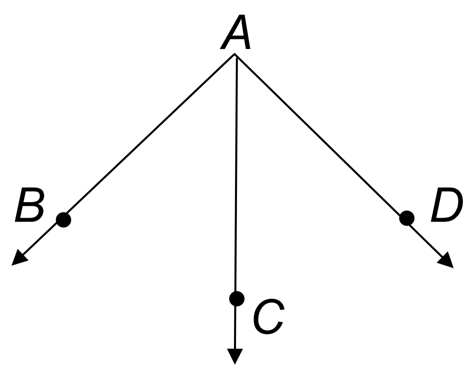


$$m\angle BAD = 120°$$

$$m\angle CAD = \frac{120°}{2}$$

$$m\angle CAD = 60°$$

Acute

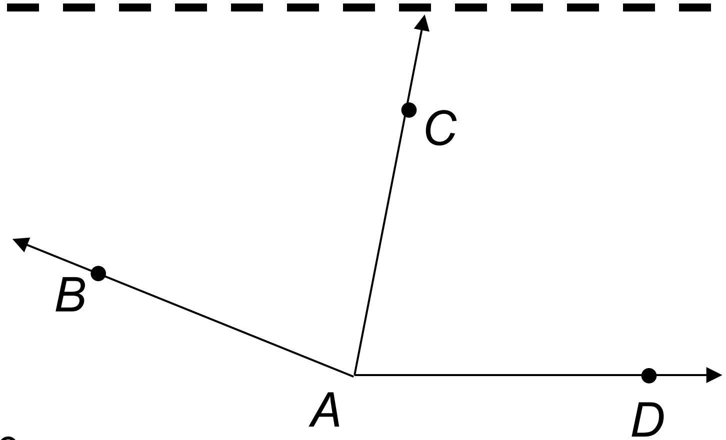


$$m\angle BAD = 92°$$

$$m\angle CAD = \frac{92°}{2}$$

$$m\angle CAD = 46°$$

Acute



$$m\angle BAD = 158°$$

$$m\angle CAD = \frac{158°}{2}$$

$$m\angle CAD = 79°$$

Acute

Example #3

In each diagram ray AC bisects obtuse angle BAD.
Classify each angle CAD as acute, right, or obtuse.
Then write and prove a conjecture about the angles formed.

Conjecture:

If an obtuse angle is bisected,
then the two newly formed congruent angles are ___**acute**___.

Statements	**Reasons**
1. $m\angle BAD < 180°$	1. Given
2. $\frac{1}{2}m\angle BAD < \frac{1}{2}(180°) = 90°$	2. Definition of Angle Bisector
3. $\frac{1}{2}m\angle BAD$ is acute.	3. Definition of Acute Angle