2.1 Inductive Reasoning

Ojectives:

- **I CAN use patterns to make conjectures.**
- **I CAN disprove geometric conjectures using counterexamples.**

Inductive Reasoning

Most learning occurs through inductive reasoning, making generalizations from observed patterns in data.

Example #1

Describe how to sketch the 4th figure. Then sketch it.

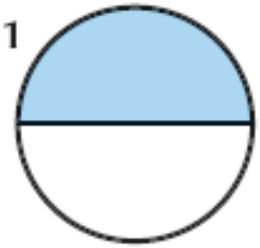


Figure 1

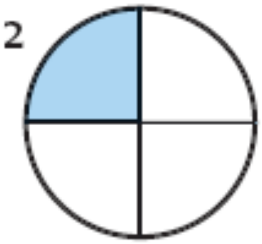


Figure 2

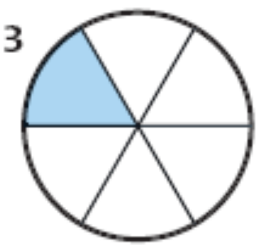


Figure 3

Each circle is divided into twice as many equal regions as the figure number. The fourth figure should be divided into eighths and the section just above the horizontal segment on the left should be shaded.


Figure 4

A Sequence…Find the next term

- 6, 17, 28, 39, … , 50
- 11, 15, 19, 23, … , 27
- 30, 26, 22, 18, … , 14
- 2, 4, 7, 11, … , 16
- 11, 16, 22, 29, … , 37

Example #2

Describe the pattern. Write the next three numbers.

$$-7, \quad -21, \quad -63, \quad -189, \ldots$$

$\times 3 \qquad \times 3 \qquad \times 3$

Multiply by 3 to get the next number in the sequence.

$$-189 \times 3 = -567$$

$$-567 \times 3 = -1701$$

$$-1701 \times 3 = -5103$$

Everyday Conjectures

- ***If*** I flip the light switch up,
  then … the light will turn on.

- ***If*** I touch the stove while it's on,
  then … I will burn my hand.

- ***If***

  then…

EXAMPLE A

A scientist dips a platinum wire into a solution containing salt (sodium chloride), passes the wire over a flame, and observes that it produces an orange-yellow flame. She does this with many other solutions that contain salt, finding that they all produce an orange-yellow flame. Make a conjecture based on her findings.

Platinum wire flame test

"If a solution contains sodium chloride,
then in a flame test it produces an orange-yellow flam

What is a conjecture?

Conjecture: conclusion made based on observation

What is inductive reasoning?

Inductive Reasoning: conjecture based on patterns

Proving conjectures TRUE is very hard.

Proving conjectures FALSE is much easier.

What is a counterexample?
How do you disprove a conjecture?

Counterexample: example that shows a conjecture is false

What are the steps for inductive reasoning?
How do you use inductive reasoning?

Steps for Inductive Reasoning

1. Find pattern.
2. Make a conjecture.
3. Test your conjecture or find a counterexample.

Example #3

Make and test a conjecture about the sum of any 3 consecutive numbers.

(Consecutive numbers are numbers that follow one after another like 3, 4, and 5.)

$$3+4+5=12=4\cdot 3$$

$$6+7+8=21=7\cdot 3$$

$$8+9+10=27=9\cdot 3$$

$$11+12+13=36=12\cdot 3$$

Conjecture:

The sum of any 3 consecutive numbers is 3 times the middle number.

$$-1+0+1=0=0\cdot 3$$

$$20+21+22=63=21\cdot 3$$

Example #4
Conjecture:
The sum of two numbers is always greater than the larger number.
True or false?

$$-2+0=-2$$

sum > larger number

$$-2<0$$

A counterexample was found, so the conjecture is false.

Inductive Reasoning
1. Look for a pattern.
2. Make a conjecture.
3. Prove the conjecture or find a counterexample.

Example #5 : Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

For every integer *n*, n^3 is positive.

Pick integers and substitute them into the expression to see if the conjecture holds.

Let $n = 1$. Since $n^3 = 1$ and $1 > 0$, the conjecture holds.

Let $n = -3$. Since $n^3 = -27$ and $-27 \leq 0$, the conjecture is false.

$n = -3$ is a counterexample.

Example 6: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

Two complementary angles are not congruent.

$45° + 45° = 90°$

If the two congruent angles both measure 45°, the conjecture is false.

Example 7: Finding a Counterexample

Show that the conjecture is false by finding a counterexample.

The monthly high temperature in Miami is never below 90°F for two months in a row.

Monthly High Temperatures (°F) in Miami, Florida											
Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
88	89	97	99	107	109	110	107	106	103	92	89

The monthly high temperatures in January and February were 88°F and 89°F, so the conjecture is false.

Example #8

Show that the conjecture is false by finding a counterexample.

For any real number x, $x^2 \geq x$.

Let $x = \frac{1}{2}$.

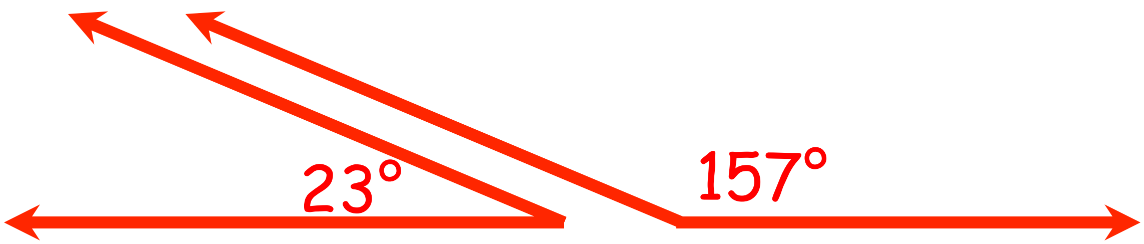
Since $\left(\frac{1}{2}\right)^2 = \frac{1}{4}$, $\frac{1}{4} \not\geq \frac{1}{2}$.

The conjecture is false.

Example #9

Show that the conjecture is false by finding a counterexample.

Supplementary angles are adjacent.



The supplementary angles are not adjacent, so the conjecture is false.

Example #10

Show that the conjecture is false by finding a counterexample.

The radius of every planet in the solar system is less than 50,000 km.

Planets' Diameters (km)							
Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune
4880	12,100	12,800	6790	143,000	121,000	51,100	49,500

Since the radius is half the diameter, the radius of Jupiter is 71,500 km and the radius of Saturn is 60,500 km. The conjecture is false.