LESSON 5.5

Properties of Parallelograms

If there is an opinion, facts will be found to support it.

JUDY SPROLES

In this lesson you will discover some special properties of parallelograms. A parallelogram is a quadrilateral whose opposite sides are parallel.

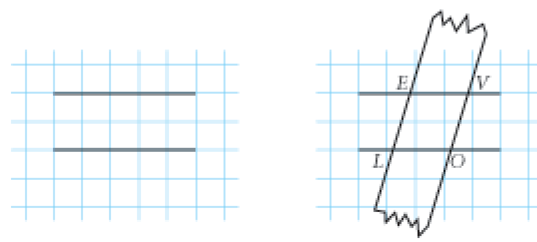
Rhombuses, rectangles, and squares all fit this definition as well. Therefore, any properties you discover for parallelograms will also apply to these other shapes. However, to be sure that your conjectures will apply to *any* parallelogram, you should investigate parallelograms that don't have any other special properties such as right angles, all congruent angles, or all congruent sides.

Investigation
Four Parallelogram Properties

You will need

- graph paper
- patty paper or a compass
- a double-edged straightedge
- a protractor

First you'll create a parallelogram.



Step 1 Step 2

Step 1 Using the lines on a piece of graph paper as a guide, draw a pair of parallel lines that are at least 6 cm apart. Using the parallel edges of your double-edged straightedge, make a parallelogram. Label your parallelogram *LOVE*.

Step 2 Let's look at the opposite angles. Measure the angles of parallelogram *LOVE*. Compare a pair of opposite angles using patty paper or your protractor.

Compare results with your group. Copy and complete the conjecture.

Parallelogram Opposite Angles Conjecture C-44

The opposite angles of a parallelogram are __?__.

Two angles that share a common side in a polygon are consecutive angles. In parallelogram *LOVE*, ∠*LOV* and ∠*EVO* are a pair of consecutive angles. The consecutive angles of a parallelogram are also related.

Step 3 Find the sum of the measures of each pair of consecutive angles in parallelogram *LOVE*.

Share your observations with your group. Copy and complete the conjecture.

Parallelogram Consecutive Angles Conjecture C-45

The consecutive angles of a parallelogram are _?_ .

Step 4 Describe how to use the two conjectures you just made to find all the angles of a parallelogram with only one angle measure given.

Step 5 Next let's look at the opposite sides of a parallelogram. With your compass or patty paper, compare the lengths of the opposite sides of the parallelogram you made.

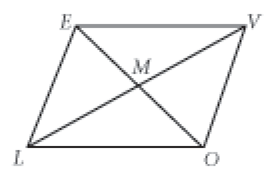
Share your results with your group. Copy and complete the conjecture.

Parallelogram Opposite Sides Conjecture C-46

The opposite sides of a parallelogram are _?_ .

Step 6 Finally, let's consider the diagonals of a parallelogram. Construct the diagonals $\overline{LV}$ and $\overline{EO}$, as shown below. Label the point where the two diagonals intersect point *M*.

Step 7 Measure *LM* and *VM*. What can you conclude about point *M*? Is this conclusion also true for diagonal $\overline{EO}$? How do the diagonals relate?



Share your results with your group. Copy and complete the conjecture.

Parallelogram Diagonals Conjecture C-47

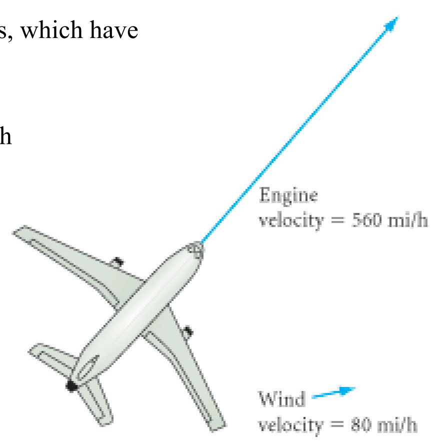
The diagonals of a parallelogram _?_ .

keymath.com/DG

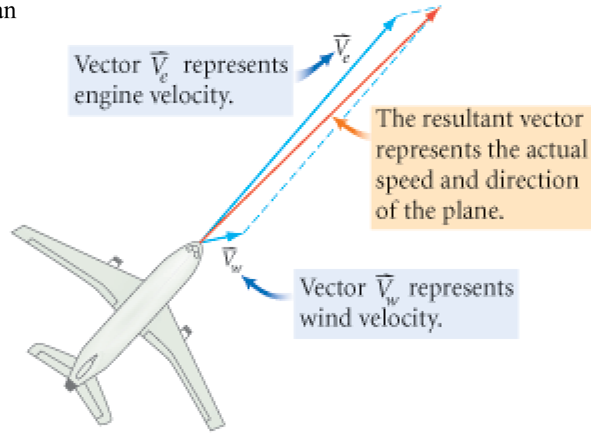
For an interactive version of this investigation, see the **Dynamic Geometry Exploration** Properties of Parallelograms at www.keymath.com/DG .

Parallelograms are used in vector diagrams, which have many applications in science. A **vector** has both magnitude and direction.

Vectors describe quantities in physics, such as velocity, acceleration, and force. You can represent a vector by drawing an arrow. The length and direction of the arrow represent the magnitude and direction of the vector. For example, a velocity vector tells you an airplane's speed and direction. The lengths of vectors in a diagram are proportional to the quantities they represent.



In many physics problems, you combine vector quantities acting on the same object. For example, the wind current and engine thrust vectors determine the velocity of an airplane. The **resultant vector** of these vectors is a single vector that has the same effect. It can also be called a **vector sum.** To find a resultant vector, make a parallelogram with the vectors as sides. The resultant vector is the diagonal of the parallelogram from the two vectors' tails to the opposite vertex.



In the diagram at right, the resultant vector shows that the wind will speed up the plane, and will also blow it slightly off course.

keymath.com/DG

For an interactive version of this diagram, see the **Dynamic Geometry Exploration** Resultant Vector at www.keymath.com/DG.

Keymath.com Practice Your Skills

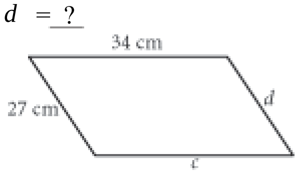
EXERCISES

You will need

Construction tools for Exercises 7 and 8

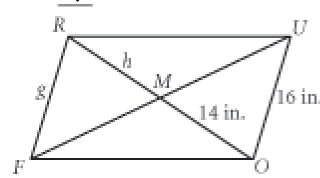
Use your new conjectures in the following exercises.
In Exercises 1–6, each figure is a parallelogram.

1. c = ?
d = ?



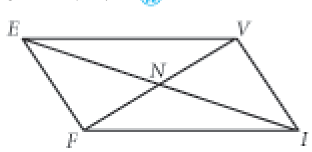
2. a = ?
b = ?

3. g = ?
h = ?

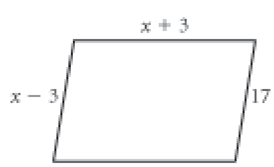


4. VF = 36 m
EF = 24 m
EI = 42 m

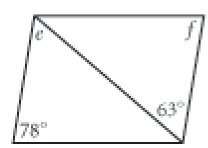
What is the perimeter of $\triangle NVI$?



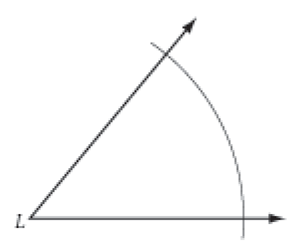
5. What is the perimeter?



6. e = ?
f = ?



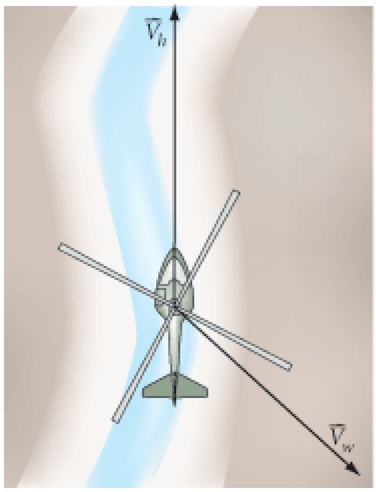
7. ***Construction*** Given side $\overline{LA}$, side $\overline{AS}$, and $\angle L$, construct parallelogram $LAST$.



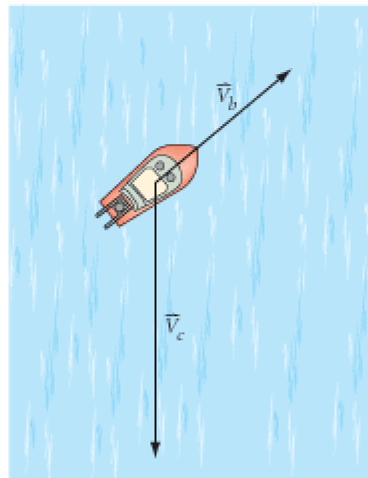
8. ***Construction*** Given side $\overline{DR}$ and diagonals $\overline{DO}$ and $\overline{PR}$, construct parallelogram *DROP*. (h)

In Exercises 9 and 10, copy the vector diagram and draw the resultant vector.

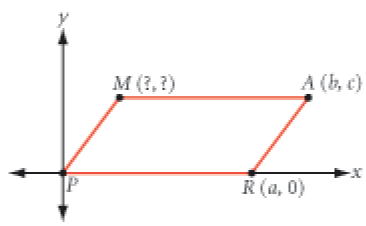
9.



10. (h)

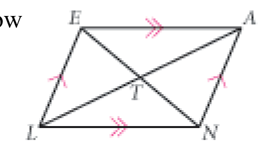


11. Find the coordinates of point *M* in parallelogram *PRAM*. (h)



12. Draw a quadrilateral. Make a copy of it. Draw a diagonal in the first quadrilateral. Draw the *other* diagonal in the duplicate quadrilateral. Cut each quadrilateral into two triangles along the diagonals. Arrange the four triangles into a parallelogram. Make a sketch showing how you did it.

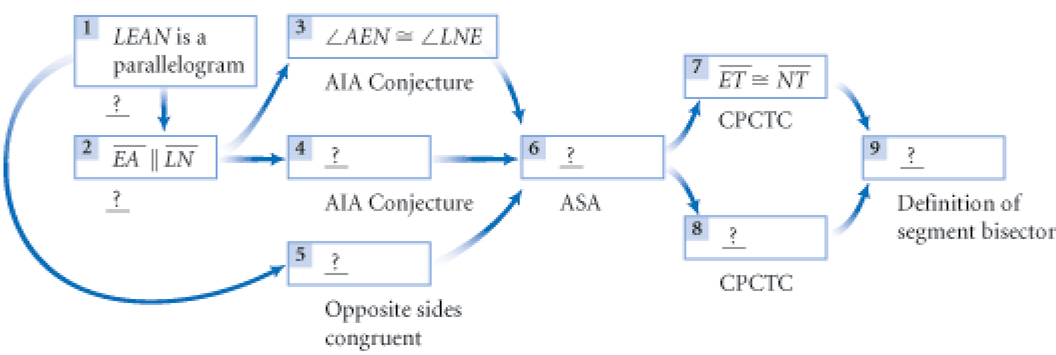
13. ***Developing Proof*** Copy and complete the flowchart to show how the Parallelogram Diagonals Conjecture follows logically from other conjectures.

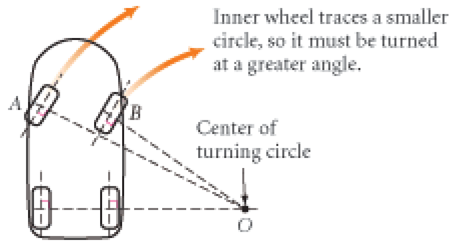


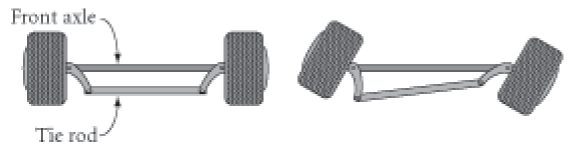
Given: *LEAN* is a parallelogram

Show: $\overline{EN}$ and $\overline{LA}$ bisect each other

Flowchart Proof







Trapezoid linkage (top view)

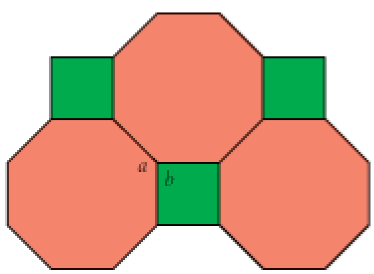
Technology CONNECTION

Quadrilateral linkages are used in mechanical design, robotics, the automotive industry, and toy making. In cars, they are used to turn each front wheel the right amount for a smooth turn.

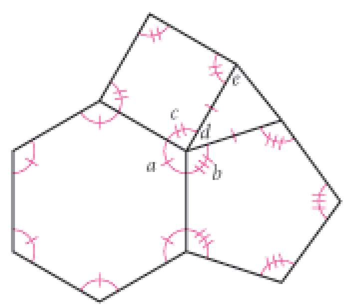
14. Study the sewing box pictured here. Sketch the box as viewed from the side, and explain why a parallelogram linkage is used.

Review

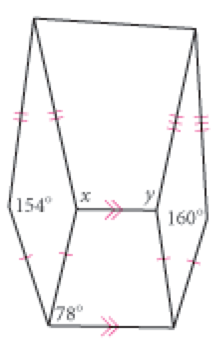
15. Find the measures of the lettered angles in this tiling of regular polygons.



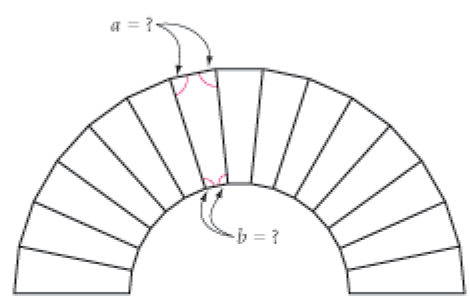
16. Trace the figure below. Calculate the measure of each lettered angle.



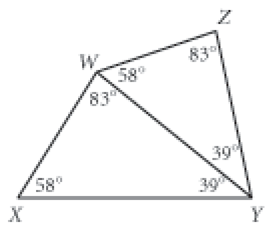
17. Find x and y. Explain.



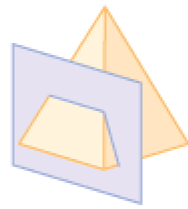
18. What is the measure of each angle in the isosceles trapezoid face of a voussoir in this 15-stone arch?



19. ***Developing Proof*** Is $\triangle XYW \cong \triangle WYZ$? Explain.



20. Sketch the section formed when this pyramid is sliced by the plane.



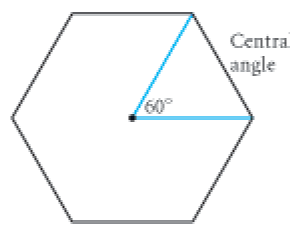
21. ***Developing Proof*** Construct two segments that bisect each other. Connect their endpoints. What type of quadrilateral is this? Draw a diagram and explain why.

22. ***Developing Proof*** Construct two intersecting circles. Connect the two centers and the two points of intersection to form a quadrilateral. What type of quadrilateral is this? Draw a diagram and explain why.

project

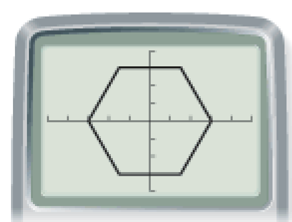
DRAWING REGULAR POLYGONS

You can draw a regular polygon's central angle by extending segments from the center of &the polygon to its consecutive vertices. For example, the measure of each central angle of a hexagon is 60°.



Using central angles, you can draw regular polygons on a graphing calculator. This is done with parametric equations, which give the *x*- and *y*-coordinates of a point in terms of a third variable, or parameter, *t*.

Set your calculator's mode to degrees and parametric. Set a friendly window with an *x*-range of −4.7 to 4.7 and a *y*-range of −3.1 to 3.1. Set a *t*-range of 0 to 360, and *t*-step of 60. Enter the equations $x = 3 \cos t$ and $y = 3 \sin t$, and graph them. You should get a hexagon.



The equations you graphed are actually the parametric equations for a circle. By using a *t*-step of 60 for *t*-values from 0 to 360, you tell the calculator to compute only six points for the circle.

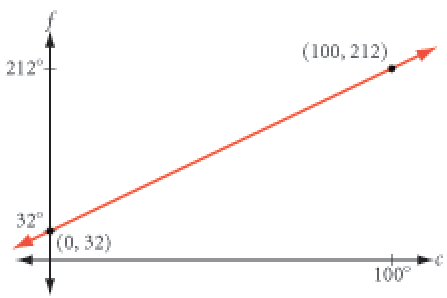
1. Choose different *t*-steps to draw different regular polygons. Find the measure of each central angle for at least three different *n*-gons.
2. What happens to the measure of each central angle as you draw polygons with more and more sides?
3. Experiment with rotating your polygons by choosing different *t*-min and *t*-max values. For example, set a *t*-range of −45 to 315, and then draw a square.
4. Explain how to draw star polygons on your calculator (see p. 266).

Your project should include

- Your answers to at least three of the questions above.
- A summary of your findings.

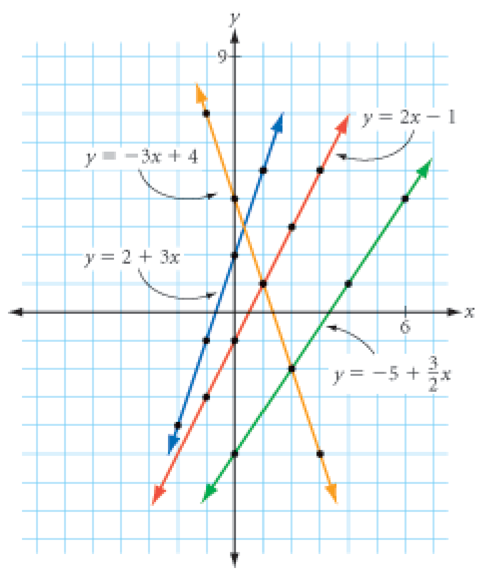
Writing Linear Equations

A linear equation is an equation whose graph is a straight line. Linear equations are useful in science, business, and many other areas. For example, the linear equation $f = 32 + \frac{9}{5}c$ gives the rule for converting a temperature from degrees Celsius, c, to degrees Fahrenheit, f. The numbers 32 and $\frac{9}{5}$ determine the graph of the equation.



Understanding how the numbers in a linear equation determine the graph can help you write a linear equation based on information about a graph.

The y-coordinate at which a graph crosses the y-axis is called the **y-intercept.** The measure of steepness is called the slope. Below are the graphs of four equations. The table gives the equation, slope, and y-intercept for each graph. How do the numbers in each equation relate to the slope and y-intercept?



Equation	Slope	y-intercept
$y = 2 + 3x$	3	2
$y = 2x - 1$	2	−1
$y = -3x + 4$	−3	4
$y = -5 + \frac{3}{2}x$	$\frac{3}{2}$	−5

In each case, the slope of the line is the coefficient of x in the equation. The y-intercept is the constant that is added to the x term.

In your algebra class, you may have learned about one of these forms of a linear equation in slope-intercept form:

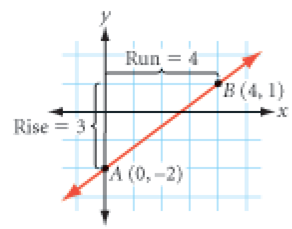
$y = mx + b$, where m is the slope and b is the y-intercept

$y = a + bx$, where a is the y-intercept and b is the slope

The only difference between these two forms is the order of the *x* term and the constant term. For example, the equation of a line with slope −3 and *y*-intercept 1 can be written as $y = -3x + 1$ or $y = 1 - 3x$.

Let's look at a few examples that show how you can apply what you have learned about the relationship between a linear equation and its graph.

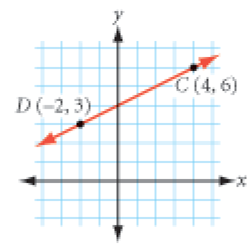
EXAMPLE A Find the equation of $\overleftrightarrow{AB}$ from its graph.



▶ Solution $\overleftrightarrow{AB}$ has *y*-intercept −2 and slope $\frac{3}{4}$, so the equation is

$$y = \frac{3}{4}x - 2$$

EXAMPLE B Given points *C*(4, 6) and *D*(−2, 3), find the equation of $\overleftrightarrow{CD}$.



▶ Solution Calculate the slope.

$$\text{slope of } \overleftrightarrow{CD} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3 - 6}{-2 - 4} = \frac{-3}{-6} = \frac{1}{2}$$

Substitute the slope into the slope-intercept form of the line, $y = mx + b$.

$$y = \frac{1}{2}x + b$$

Use either given point, say (4, 6), to form an equation with one variable and solve.

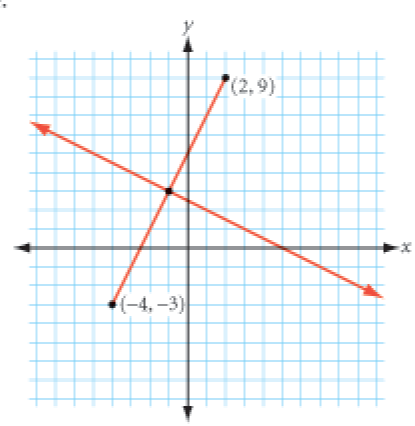
$$6 = \frac{1}{2}(4) + b$$

$$6 = 2 + b$$

$$4 = b$$

Thus, the equation for $\overleftrightarrow{CD}$ is $y = \frac{1}{2}x + 4$.

EXAMPLE C Find the equation of the perpendicular bisector of the segment with endpoints (2, 9) and (−4, −3).



▶ Solution The perpendicular bisector passes through the midpoint of the segment.

$$x = \frac{2 + (-4)}{2} = \frac{-2}{2} = -1$$

$$y = \frac{9 + (-3)}{2} = \frac{6}{2} = 3$$

midpoint is (−1, 3)

Calculate the slope of the segment.

$$\text{slope of segment} = \frac{-3-9}{-4-2} = \frac{-12}{-6} = 2$$

The slope of the perpendicular bisector is the opposite reciprocal of the slope of the segment.

$$\text{slope of perpendicular bisector} = -\frac{1}{2}$$

To find the y-intercept of the perpendicular bisector, either use the method in Example B or the slope formula. The slope between any point (x, y) on the perpendicular bisector and the calculated midpoint $(-1, 3)$ must be $-\frac{1}{2}$.

$$\frac{y-3}{x-(-1)} = -\frac{1}{2}$$

Solve this equation for y to find the equation of the perpendicular bisector.

$$\frac{y-3}{x+1} = \frac{-1}{2}$$

$$(x+1)\left(\frac{y-3}{x+1}\right) = (x+1)\left(\frac{-1}{2}\right)$$

$$y-3 = \frac{-x-1}{2}$$

$$2(y-3) = 2\left(\frac{-x-1}{2}\right)$$

$$2y-6 = -x-1$$

$$2y = -x+5$$

$$y = -\frac{1}{2}x + 2\frac{1}{2}$$

EXERCISES

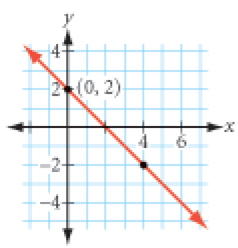
Practice Your Skills

In Exercises 1–3, graph each linear equation.

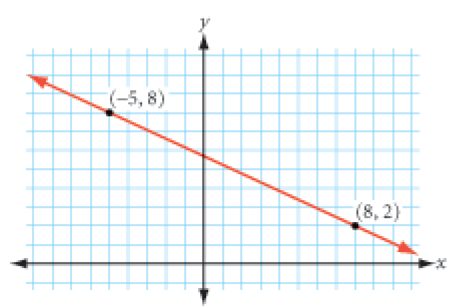
1. $y = 1 - 2x$ **2.** $y = \frac{4}{3}x + 4$ **3.** $2y - 3x = 12$

Write an equation for each line in Exercises 4 and 5.

4.



5.



In Exercises 6–8, write an equation for the line through each pair of points.

6. (1, 2), (3, 4) **7.** (1, 2), (3, −4) **8.** (−1, −2), (−6, −4)

9. The math club is ordering printed T-shirts to sell for a fundraiser. The T-shirt company charges \$80 for the set-up fee and \$4 for each printed T-shirt. Using x for the number of shirts the club orders, write an equation for the total cost of the T-shirts.

10. Write an equation for the line with slope −3 that passes through the midpoint of the segment with endpoints (3, 4) and (11, 6).

11. Write an equation for the line that is perpendicular to the line $y = 4x + 5$ and that passes through the point (0, −3).

For Exercises 12–14, the coordinates of the vertices of $\triangle WHY$ are $W(0, 0)$, $H(8, 3)$, and $Y(2, 9)$.

12. Find the equation of the line containing median $\overline{WO}$.

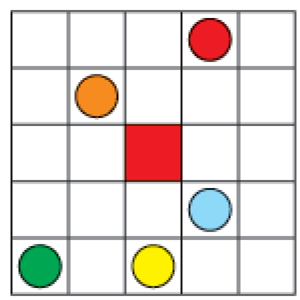
13. Find the equation of the perpendicular bisector of side $\overline{HY}$.

14. Find the equation of the line containing altitude $\overline{HT}$.

IMPROVING YOUR REASONING SKILLS

Lunar Lockout

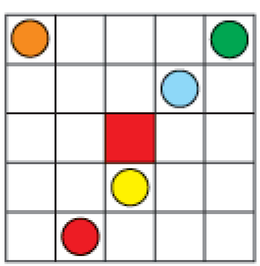
The goal of this puzzle is to move the red piece into the center square. All of the pieces move according to these two rules:



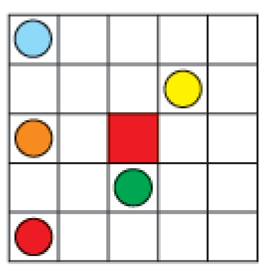
- A piece can move only horizontally or vertically, not diagonally.
- A piece continues to move until its path is blocked by another piece.

A piece can't move in a direction where there is no other piece to block its path. In the board at right, for example, you can move the blue piece up until it is stopped by the red piece, but you can't move it in any other direction. One possible solution to this puzzle is this sequence of moves: green right, green up, red down, red left.

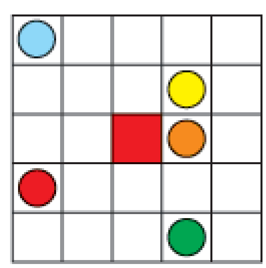
Try the puzzles below.



Beginner Puzzle



Intermediate Puzzle



Advanced Puzzle

Find links to online versions of Lunar Lockout and other games and puzzles by ThinkFun™ at www.keymath.com/DG .