Student Web Links @ Keymath.com

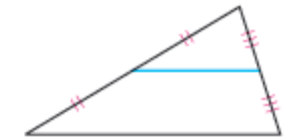
LESSON 5.4

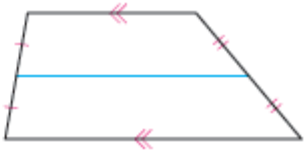
Properties of Midsegments

Research is formalized curiosity. It is poking and prying with a purpose.
ZORA NEALE HURSTON

As you learned in Chapter 3, the segment connecting the midpoints of two sides of a triangle is a midsegment of the triangle. The segment connecting the midpoints of the two nonparallel sides of a trapezoid is also called the midsegment of the trapezoid.

In this lesson you will discover special properties of midsegments.



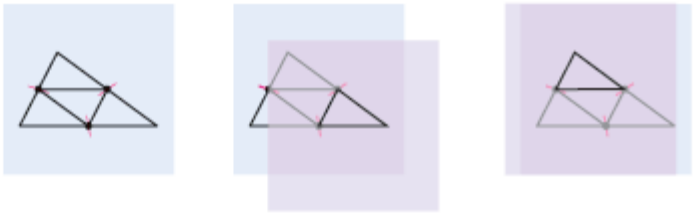


Investigation 1

Triangle Midsegment Properties

You will need
- patty paper
- a straightedge

In this investigation you will discover two properties of triangle midsegments. Each person in your group can investigate a different triangle.



Step 1 Step 2 Step 3

Step 1 Draw a triangle on a piece of patty paper. Pinch the patty paper to locate midpoints of the sides. Draw the midsegments.You should now have four small triangles.

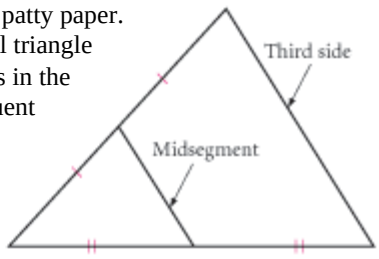
Step 2 Place a second piece of patty paper over the first and copy one of the four triangles.

Step 3 Compare all four triangles by sliding the copy of one small triangle over the other three triangles. Compare your results with the results of your group. Copy and complete the conjecture.

Three Midsegments Conjecture C-41

The three midsegments of a triangle divide it into ? .

Step 4 Mark all the congruent angles on the original patty paper. If you find it too cluttered, redraw the original triangle on regular paper with just one midsegment, as in the diagram at right, and then mark all the congruent angles. Using the Corresponding Angles Conjecture or its converse, what conclusions can you make about a midsegment and the large triangle's third side?



Step 5 Compare the length of the midsegment to the large triangle's third side. How do they relate? Copy and complete the conjecture.

Triangle Midsegment Conjecture C-42

A midsegment of a triangle is ? to the third side and ? the length of ?.

In the next investigation you will discover two properties of the midsegment of a trapezoid.

Investigation 2
Trapezoid Midsegment Properties

You will need
- patty paper
- a straightedge

Each person in your group can investigate a different trapezoid. Make sure you draw the two bases perfectly parallel.

Step 1 Step 2 Step 3

Step 1 Draw a small trapezoid on the left side of a piece of patty paper. Pinch the paper to locate the midpoints of the nonparallel sides. Draw the midsegment.

Step 2 Label the angles as shown. Place a second piece of patty paper over the first and copy the trapezoid and its midsegment.

Step 3 Compare the trapezoid's base angles with the corresponding angles at the midsegment by sliding the copy up over the original.

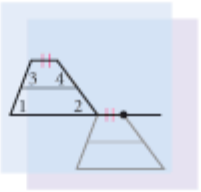
Step 4 Are the corresponding angles congruent? What can you conclude about the midsegment and the bases? Compare your results with the results of other students.

The midsegment of a triangle is half the length of the third side. How does the length of the midsegment of a trapezoid compare to the lengths of the two bases? Let's investigate.

Step 5 On the original trapezoid, extend the longer base to the right by at least the length of the shorter base.

Step 6 Slide the second patty paper under the first. Show the sum of the lengths of the two bases by marking a point on the extension of the longer base.

Step 5



Step 6

Step 7

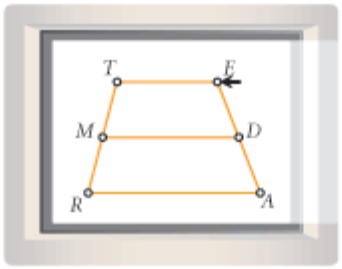
Step 7 How many times does the midsegment fit onto the segment representing the sum of the lengths of the two bases? What do you notice about the length of the midsegment and the sum of the lengths of the two bases?

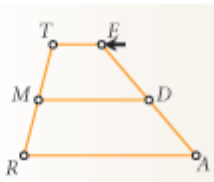
Step 8 Combine your conclusions from Steps 4 and 7 and complete this conjecture.

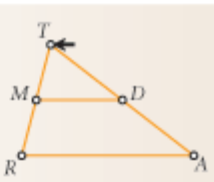
Trapezoid Midsegment Conjecture C-43

The midsegment of a trapezoid is ? to the bases and is equal in length to ?.

What happens if one base of the trapezoid shrinks to a point? Then the trapezoid collapses into a triangle, the midsegment of the trapezoid becomes a midsegment of the triangle, and the Trapezoid Midsegment Conjecture becomes the Triangle Midsegment Conjecture. Do both of your midsegment conjectures work for the last figure?







keymath.com/DG

[▸ For an interactive version of this sketch, see the **Dynamic Geometry Exploration** Properties of the Midsegment of a Trapezoid at www.keymath.com/DG .◂]

EXERCISES

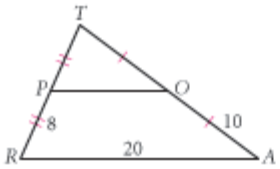
Keymath.com Practice Your Skills

You will need

Construction tools for Exercises 9 and 18

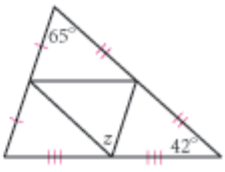
1. How many midsegments does a triangle have? A trapezoid have?

2. What is the perimeter of △*TOP*? (h)

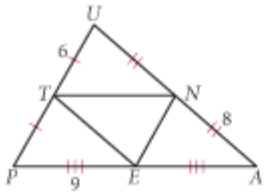


3. x = ?
y = ?

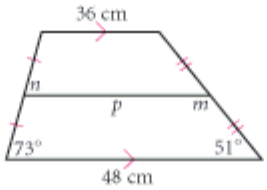
4. z = ? (h)



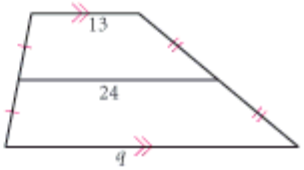
5. What is the perimeter of $\triangle TEN$?



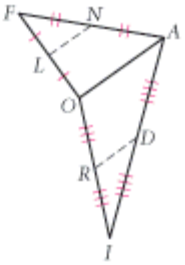
6. m = ?
n = ?
p = ?



7. q = ?



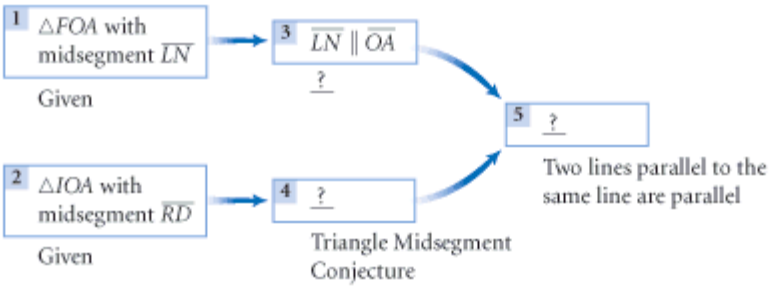
8. ***Developing Proof*** Copy and complete the flowchart to show that $\overline{LN} \parallel \overline{RD}$.



Given: Midsegment $\overline{LN}$ in $\triangle FOA$
Midsegment $\overline{RD}$ in $\triangle IOA$

Show: $\overline{LN} \parallel \overline{RD}$

Flowchart Proof



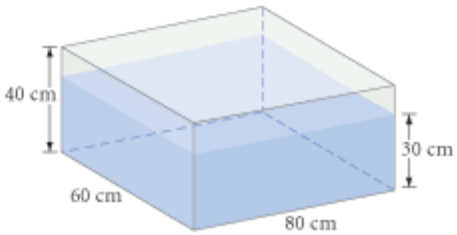
9. ***Construction*** When you connected the midpoints of the three sides of a triangle in Investigation 1, you created four congruent triangles. Draw a quadrilateral on patty paper and pinch the paper to locate the midpoints of the four sides. Connect the midpoints to form a quadrilateral. What special type of quadrilateral do you get when you connect the midpoints? Use the Triangle Midsegment Conjecture to explain your answer. (h)

10. Deep in a tropical rain forest, archaeologist Ertha Diggs and her assistant researchers have uncovered a square-based truncated pyramid (a square pyramid with the top part removed). The four lateral faces are isosceles trapezoids. A line of darker mortar runs along the midsegment of each lateral face. Ertha and her co-workers make some measurements and find that one of these midsegments measures 41 meters, and each bottom base measures 52 meters. Now that they have this information, Ertha and her team can calculate the length of the top base without having to climb up and measure it. Can you? What is the length of the top edge? How do you know?

11. Ladie and Casey pride themselves on their estimation skills and take turns estimating distances. Casey claims that two large redwood trees visible from where they are sitting are 180 feet apart, and Ladie says they are 275 feet apart.

The problem is, they can't measure the distance to see whose estimate is better, because their cabin is located between the trees. All of a sudden, Ladie recalls her geometry: "Oh yeah, the Triangle Midsegment Conjecture!" She collects a tape measure, a hammer, and some wooden stakes. What is she going to do?

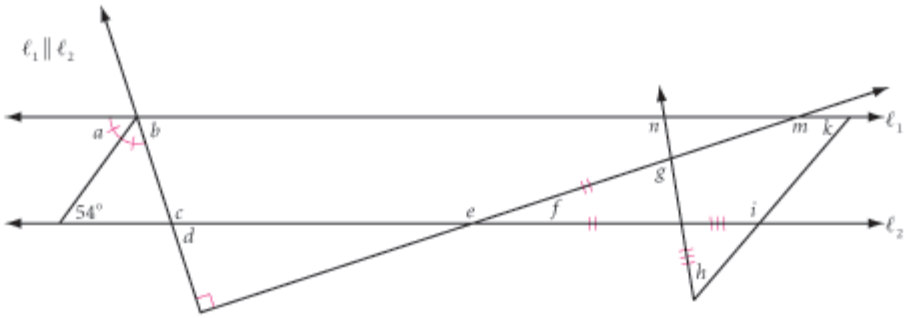
Review



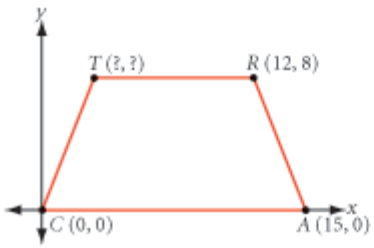
12. The 40-by-60-by-80 cm sealed rectangular container shown at right is resting on its largest face. It is filled with a liquid to a height of 30 cm. Sketch the container resting on its smallest face. Show the height of the liquid in this new position. Explain your method.

13. ***Developing Proof*** Write the converse of this statement: If exactly one diagonal bisects a pair of opposite angles of a quadrilateral, then the quadrilateral is a kite. Is the converse true? Is the original statement true? If either conjecture is not true, sketch a counterexample.

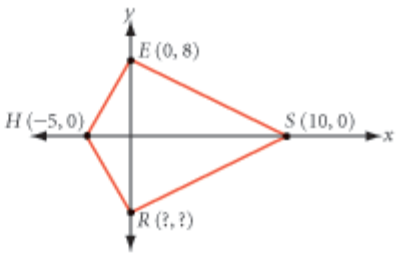
14. ***Developing Proof*** Trace the figure below. Calculate the measure of each lettered angle. Explain how you determined the measures *h* and *k*.



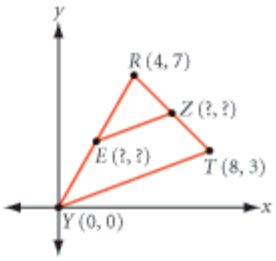
15. *CART* is an isosceles trapezoid. What are the coordinates of point *T*?



16. *HRSE* is a kite. What are the coordinates of point *R*?



17. Find the coordinates of midpoints E and Z. Show that the slope of the line containing midsegment $\overline{EZ}$ is equal to the slope of the line containing $\overline{YT}$.



18. ***Construction*** Use the kite properties you discovered in Lesson 5.3 to construct kite $FRNK$ given diagonals $\overline{RK}$ and $\overline{FN}$ and side $\overline{NK}$. Is there only one solution?

project

BUILDING AN ARCH

In this project you'll design and build your own Roman arch.

Horseshoe Arch

Basket Arch

Tudor Arch

Lancet Arch

Arches can have a simple semicircular shape, or a pointed "broken arch" shape.

In arch construction, a wooden support holds the voussoirs in place until the keystone is placed (see arch diagram on page 273). It's said that when the Romans made an arch, they would make the architect stand under it while the wooden support was removed. That was one way to be sure architects carefully designed arches that wouldn't fall!

What size arch would you like to build? Decide the dimensions of the opening, the thickness of the arch, and the number of voussoirs. Decide on the materials you will use. You should have your trapezoid and your materials approved by your group or your teacher before you begin construction.

Your project should include

- A scale diagram that shows the exact size and angle of the voussoirs and the keystone.
- A template for your voussoirs.
- Your arch.

The arches in this Roman aqueduct, above the Gard River in France, are typical of arches you can find throughout regions that were once part of the Roman Empire. An arch can carry a lot of weight, yet it also provides an opening. The abutments on the sides of the arch keep the arch from spreading out and falling down.