LESSON 5.3

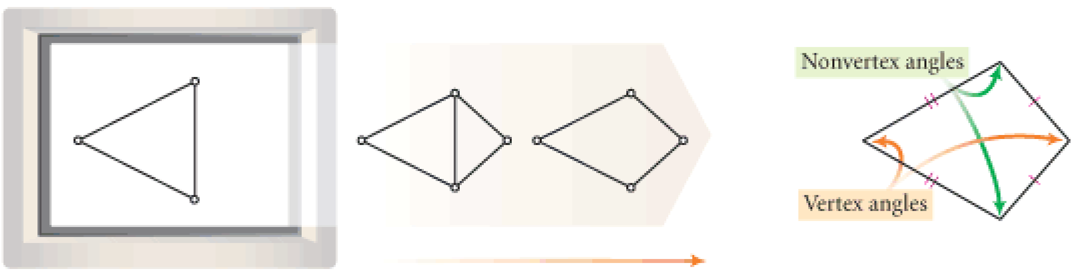
Kite and Trapezoid Properties

Imagination is the highest kite we fly.

LAUREN BACALL

Recall that a **kite** is a quadrilateral with exactly two distinct pairs of congruent consecutive sides.

If you construct two different isosceles triangles on opposite sides of a common base and then remove the base, you have constructed a kite. In an isosceles triangle, the vertex angle is the angle between the two congruent sides. Therefore, let's call the two angles between each pair of congruent sides of a kite the **vertex angles** of the kite. Let's call the other pair the **nonvertex angles.**



[For an interactive version of this sketch, see the **Dynamic Geometry Exploration** Properties of Kites at www.keymath.com/DG .]

keymath.com/DG

A kite also has one line of reflectional symmetry, just like an isosceles triangle. You can use this property to discover other properties of kites. Let's investigate.

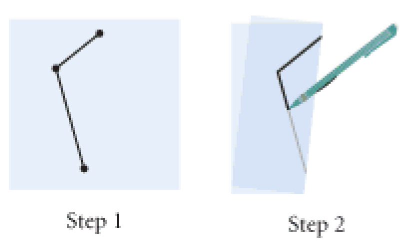
Investigation 1
What Are Some Properties of Kites?

You will need
- patty paper
- a straightedge

In this investigation you will look at angles and diagonals in a kite to see what special properties they have.

Step 1 On patty paper, draw two connected segments of different lengths, as shown. Fold through the endpoints and trace the two segments on the back of the patty paper.

Step 2 Compare the size of each pair of opposite angles in your kite by folding an angle onto the opposite angle. Are the vertex angles congruent? Are the nonvertex angles congruent? Share your observations with others near you and complete the conjecture.

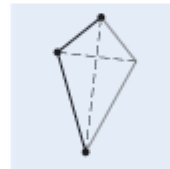


© 2008 Key Curriculum Press

Kite Angles Conjecture C-34

The _?_ angles of a kite are _?_.

Step 3 Draw the diagonals. How are the diagonals related? Share your observations with others in your group and complete the conjecture.



Kite Diagonals Conjecture C-35

The diagonals of a kite are _?_.

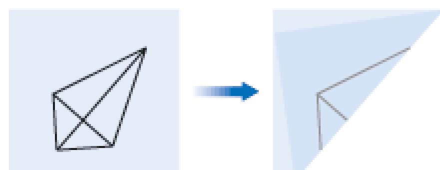
What else seems to be true about the diagonals of kites?

Step 4 Compare the lengths of the segments on both diagonals. Does either diagonal bisect the other? Share your observations with others near you. Copy and complete the conjecture.

Kite Diagonal Bisector Conjecture C-36

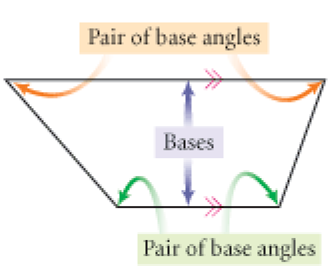
The diagonal connecting the vertex angles of a kite is the _?_ of the other diagonal.

Step 5 Fold along both diagonals. Does either diagonal bisect any angles? Share your observations with others and complete the conjecture.



Kite Angle Bisector Conjecture C-37

The _?_ angles of a kite are _?_ by a _?_.



You will prove the Kite Diagonal Bisector Conjecture and the Kite Angle Bisector Conjecture as exercises after this lesson.

Let's move on to trapezoids. Recall that a **trapezoid** is a quadrilateral with exactly one pair of parallel sides.

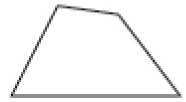
In a trapezoid the parallel sides are called **bases.** A pair of angles that share a base as a common side are called **base angles.**

In the next investigation you will discover some properties of trapezoids.

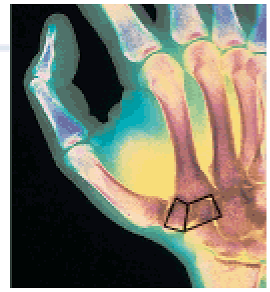
© 2008 Key Curriculum Press

Science CONNECTION

A *trapezium* is a quadrilateral with *no* two sides parallel. The words *trapezoid* and *trapezium* come from the Greek word *trapeza*, meaning table. There are bones in your wrists that anatomists call trapezoid and trapezium because of their geometric shapes.



Trapezium



Investigation 2
What Are Some Properties of Trapezoids?

You will need

- a double-edged straightedge
- a protractor
- a compass

This is a view inside a deflating hot-air balloon. Notice the trapezoidal panels that make up the balloon.

Step 1 Use the two edges of your straightedge to draw parallel segments of unequal length. Draw two nonparallel sides connecting them to make a trapezoid.

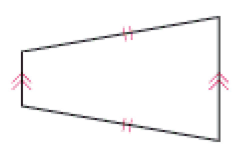
Step 2 Use your protractor to find the sum of the measures of each pair of consecutive angles between the parallel bases. What do you notice about this sum? Share your observations with your group.

Step 3 Copy and complete the conjecture.

Trapezoid Consecutive Angles Conjecture C-38

The consecutive angles between the bases of a trapezoid are __?__.

Recall from Chapter 3 that a trapezoid whose two nonparallel sides are the same length is called an **isosceles trapezoid**. Next, you will discover a few properties of isosceles trapezoids.

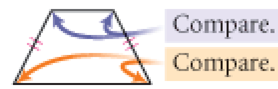


© 2008 Key Curriculum Press

Like kites, isosceles trapezoids have one line of reflectional symmetry. Through what points does the line of symmetry pass?

Step 4 Use both edges of your straightedge to draw parallel lines. Using your compass, construct two congruent, nonparallel segments. Connect the four segments to make an isosceles trapezoid.

Step 5 Measure each pair of base angles. What do you notice about the pair of base angles in each trapezoid? Compare your observations with others near you.



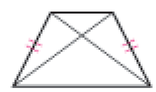
Step 6 Copy and complete the conjecture.

Isosceles Trapezoid Conjecture C-39

The base angles of an isosceles trapezoid are _?_ .

What other parts of an isosceles trapezoid are congruent? Let's continue.

Step 7 Draw both diagonals. Compare their lengths. Share your observations with others near you.

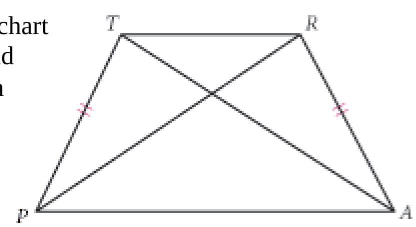


Step 8 Copy and complete the conjecture.

Isosceles Trapezoid Diagonals Conjecture C-40

The diagonals of an isosceles trapezoid are _?_ .

Developing Proof As a group, write a flowchart proof that shows how the Isosceles Trapezoid Diagonals Conjecture follows logically from the Isosceles Trapezoid Conjecture. Use the diagram at right and a method for making the triangles easier to see. If you need help, see page 232, Example B.



Keymath.com Practice Your Skills

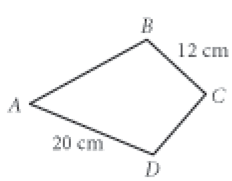
EXERCISES

You will need

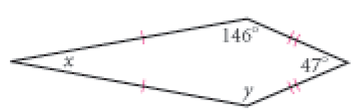
Construction tools for Exercises 14–16

Use your new conjectures to find the missing measures.

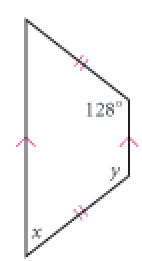
1. *ABCD* is a kite.
perimeter = _?_



2. x = _?_
y = _?_



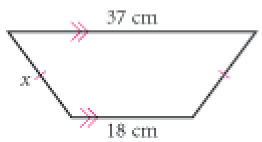
3. x = _?_
y = _?_



© 2008 Key Curriculum Press

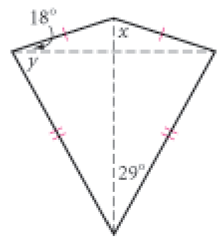
4. $x = \underline{?}$

perimeter = 85cm



5. $x = \underline{?}$

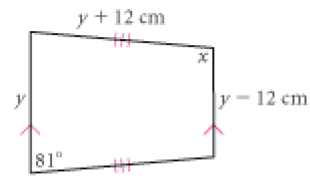
$y = \underline{?}$



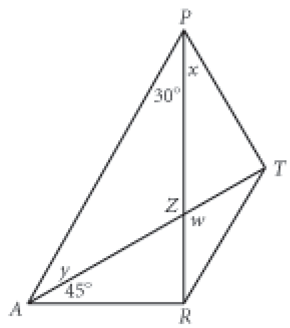
6. $x = \underline{?}$

$y = \underline{?}$

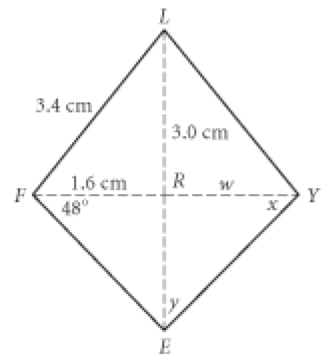
perimeter = 164 cm



7. *ARTP* is an isosceles trapezoid with $RA = PT$. Find *w*, *x*, and *y*.



8. *FLYE* is a kite with $FL = LY$. Find *w*, *x*, and *y*.

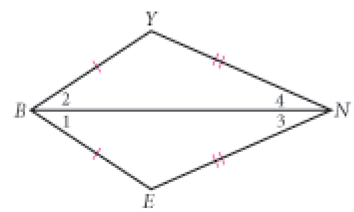


9. Copy and complete the flowchart to show how the Kite Angle Bisector Conjecture follows logically from one of the triangle congruence conjectures.

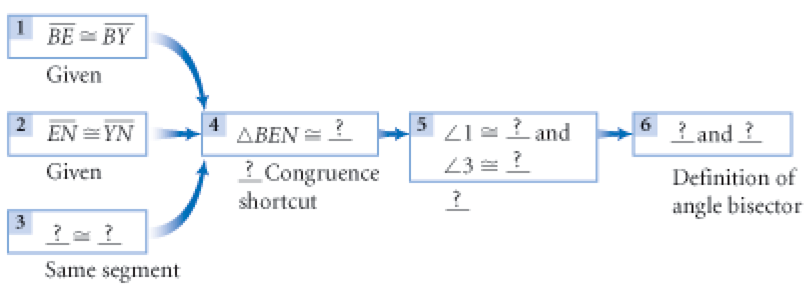
Given: Kite *BENY* with $\overline{BE} \cong \overline{BY}$, $\overline{EN} \cong \overline{YN}$

Show: $\overline{BN}$ bisects $\angle B$

$\overline{BN}$ bisects $\angle N$



Flowchart Proof



10. Write a paragraph proof or flowchart proof of the Kite Diagonal Bisector Conjecture. Either show how it follows logically from the Kite Angle Bisector Conjecture that you just proved, or how it follows logically from the Converse of the Perpendicular Bisector Conjecture.

11. Sketch and label kite *KITE* with vertex angles $\angle K$ and $\angle T$ and $KI > TE$. Which angles are congruent?

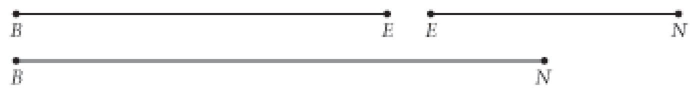
 © 2008 Key Curriculum Press

12. Sketch and label trapezoid *QUIZ* with one base $\overline{QU}$. What is the other base? Name the two pairs of base angles.

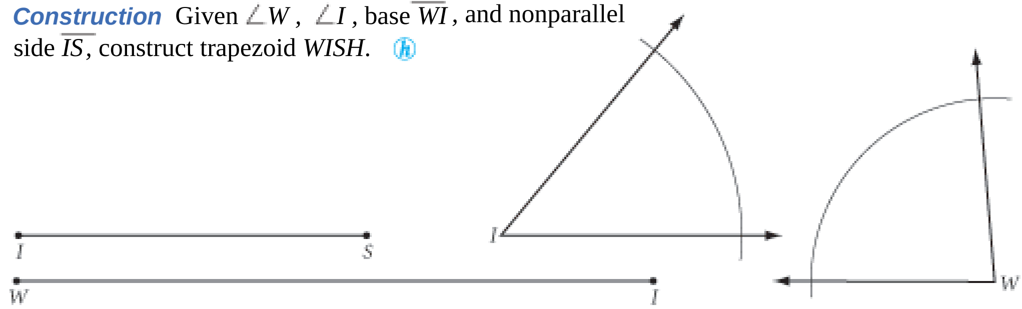
13. Sketch and label isosceles trapezoid *SHOW* with one base $\overline{SH}$. What is the other base? Name the two pairs of base angles. Name the two sides of equal length.

In Exercises 14–16, use the properties of kites and trapezoids to construct each figure. You may use either patty paper or a compass and a straightedge.

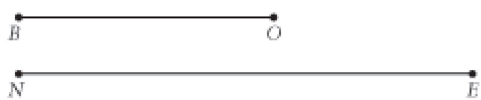
14. ***Construction*** Construct kite *BENF* given sides $\overline{BE}$ and $\overline{EN}$ and diagonal $\overline{BN}$. How many different kites are possible?



15. ***Construction*** Given $\angle W$, $\angle I$, base $\overline{WI}$, and nonparallel side $\overline{IS}$, construct trapezoid *WISH*. ⓗ

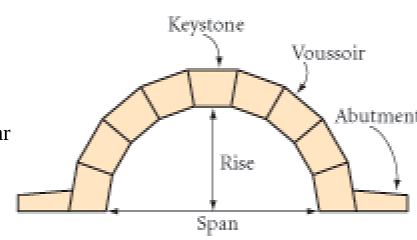


16. ***Construction*** Construct a trapezoid *BONE* with $\overline{BO} \parallel \overline{NE}$. How many different trapezoids can you construct?



Architecture CONNECTION

The Romans used the classical arch design in bridges, aqueducts, and buildings in the early centuries of the Common Era. The classical semicircular arch is really half of a regular polygon built with wedge-shaped blocks whose faces are isosceles trapezoids. Each block supports the blocks surrounding it.

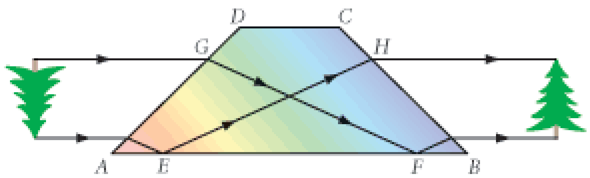


17. ***Application*** The inner edge of the arch in the diagram above right is half of a regular 18-gon. Calculate the measures of all the angles in the nine isosceles trapezoids making up the arch. Then use your geometry tools to accurately draw a nine-stone arch like the one shown.

This carton is shaped like an isosceles trapezoid block, like the voussoirs used in the arch above.

© 2008 Key Curriculum Press

18. The figure below shows the path of light through a trapezoidal prism and how an image is inverted. For the prism to work as shown, the trapezoid must be isosceles, $\angle AGF$ must be congruent to $\angle BHE$, and $\overline{GF}$ must be congruent to $\overline{EH}$. Show that if these conditions are met, then $\overline{AG}$ will be congruent to $\overline{BH}$. ⓗ

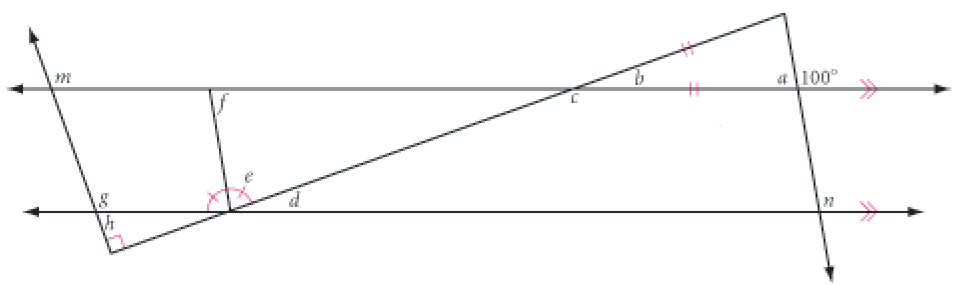


Science CONNECTION

The magnifying lenses of binoculars invert the objects you view through them, so trapezoidal prisms are used to flip the inverted images right-side-up again.

Review

19. ***Developing Proof*** Trace the figure below. Calculate the measure of each lettered angle. Explain how you determined measures *e* and *g*.



IMPROVING YOUR REASONING SKILLS

How Did the Farmer Get to the Other Side?

A farmer was taking her pet rabbit, a basket of prize-winning baby carrots, and her small—but hungry—rabbit-chasing dog to town. She came to a river and realized she had a problem. The little boat she found tied to the pier was big enough to carry only herself and one of the three possessions. She couldn't leave her dog on the bank with the little rabbit (the dog would frighten the poor rabbit), and she couldn't leave the rabbit alone with the carrots (the rabbit would eat all the carrots). But she still had to figure out how to cross the river safely with one possession at a time. How could she move back and forth across the river to get the three possessions safely to the other side?

© 2008 Key Curriculum Press