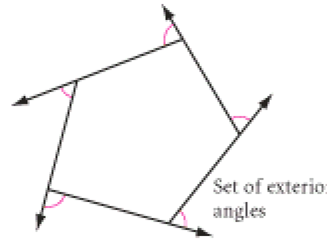
LESSON 5.2

Exterior Angles of a Polygon

If someone had told me I would be Pope someday, I would have studied harder.
POPE JOHN PAUL I

In Lesson 5.1, you discovered a formula for the sum of the measures of the *interior* angles of any convex polygon. In this lesson you will discover a formula for the sum of the measures of the *exterior* angles of a convex polygon.



Best known for her participation in the Dada Movement, German artist Hannah Hoch (1889–1978) painted *Emerging Order* in the Cubist style. Do you see any examples of exterior angles in the painting?

Investigation

Is There an Exterior Angle Sum?

You will need
- a straightedge
- a protractor

Let's use some inductive and deductive reasoning to find the exterior angle measures in a polygon.

Each person in your group should draw the same kind of polygon for Steps 1–5.

Step 1 Draw a large polygon. Extend its sides to form a set of exterior angles.

Step 2 Measure all the *interior* angles of the polygon except one. Use the Polygon Sum Conjecture to calculate the measure of the remaining interior angle. Check your answer using your protractor.

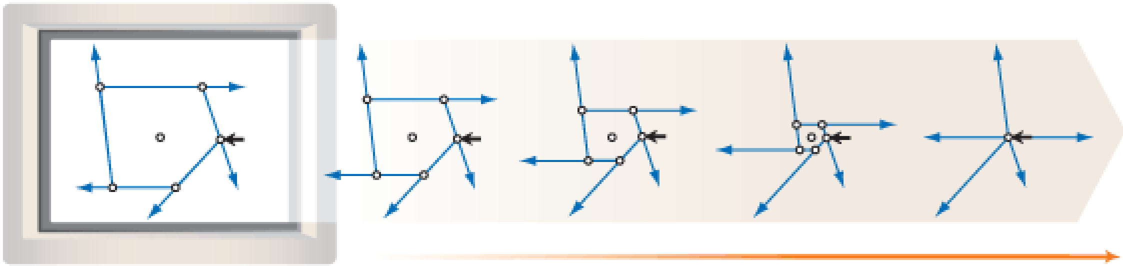
Step 3 Use the Linear Pair Conjecture to calculate the measure of each exterior angle.

Step 4 Calculate the sum of the measures of the exterior angles. Share your results with your group members.

Step 5 Repeat Steps 1–4 with different kinds of polygons, or share results with other groups. Make a table to keep track of the number of sides and the sum of the exterior angle measures for each kind of polygon. Find a formula for the sum of the measures of a polygon's exterior angles.

Exterior Angle Sum Conjecture C-32

For any polygon, the sum of the measures of a set of exterior angles is ? .



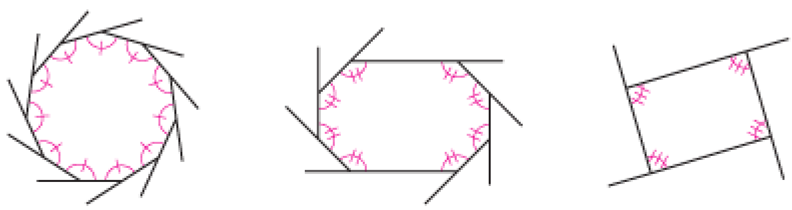
[▶ For an interactive version of this sketch, see the **Dynamic Geometry Exploration** The Exterior Angle Sum of a Polygon at www.keymath.com/DG . ◀]

keymath.com/DG

Step 6 Study the software construction above. Explain how it demonstrates the Exterior Angle Sum Conjecture.

Step 7 Using the Polygon Sum Conjecture, write a formula for the measure of each interior angle in an equiangular polygon.

Step 8 Using the Exterior Angle Sum Conjecture, write the formula for the measure of each exterior angle in an equiangular polygon.



Step 9 Using your results from Step 8, you can write the formula for an interior angle of an equiangular polygon a different way. How do you find the measure of an interior angle if you know the measure of its exterior angle? Complete the next conjecture.

Equiangular Polygon Conjecture C-33

You can find the measure of each interior angle of an equiangular *n*-gon by using either of these formulas: ? or ? .

Keymath.com Practice Your Skills

EXERCISES

1. What is the sum of the measures of the exterior angles of a decagon?

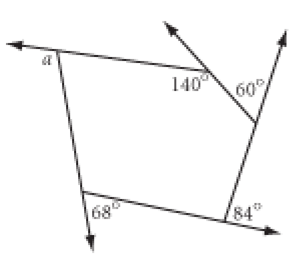
2. What is the measure of an exterior angle of an equiangular pentagon? An equiangular hexagon?

3. How many sides does a regular polygon have if each exterior angle measures 24°?

4. How many sides does a polygon have if the sum of its interior angle measures is 7380°?

In Exercises 5–10, use your new conjectures to calculate the measure of each lettered angle.

5.



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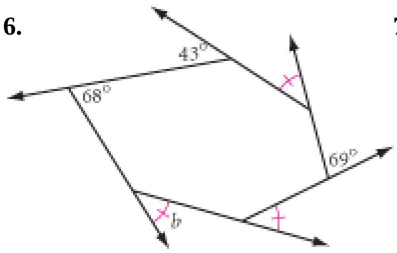

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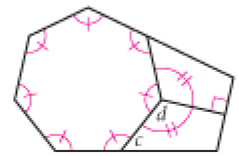

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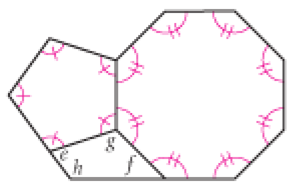

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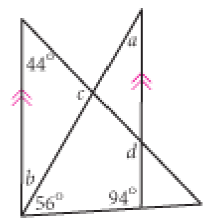

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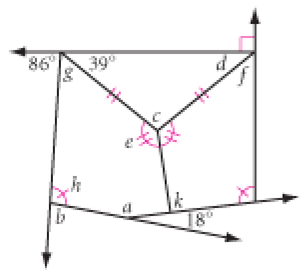
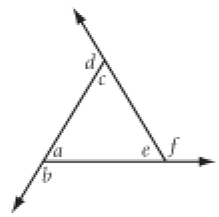


11. ***Developing Proof*** Complete this flowchart proof of the Exterior Angle Sum Conjecture for a triangle.



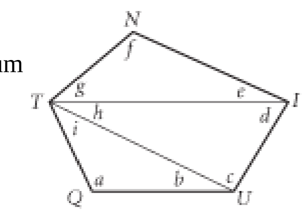
Flowchart Proof

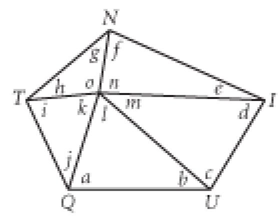
1. $a + b = 180°$ — ?
2. $c + d = 180°$ — ?
3. $e + f = 180°$ — ?
4. $a + b + c + d + e + f =$? ° — Addition property of equality
5. $a + c + e =$? ° — ?
6. $b + d + f =$? ° — Subtraction property of equality

12. Is there a maximum number of obtuse exterior angles that any polygon can have? If so, what is the maximum? If not, why not? Is there a minimum number of acute interior angles that any polygon must have? If so, what is the minimum? If not, why not?

Review

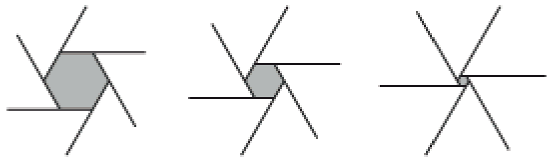
13. ***Developing Proof*** Prove the Pentagon Sum Conjecture using either method discussed in the last lesson. Use whichever diagram at right corresponds with your method. If you need help, see page 261, Exercise 17.



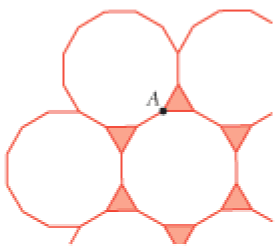


Technology CONNECTION

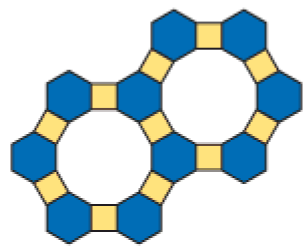
The aperture of a camera is an opening shaped like a regular polygon surrounded by thin sheets that form a set of exterior angles. These sheets move together or apart to close or open the aperture, limiting the amount of light passing through the camera's lens. How does the sequence of closing apertures shown below demonstrate the Exterior Angle Sum Conjecture? Does the number of sides make a difference in the opening and closing of the aperture?



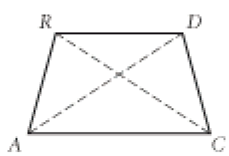
14. Name the regular polygons that appear in the tiling below. Find the measures of the angles that surround point A in the tiling.



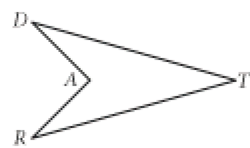
15. Name the regular polygons that appear in the tiling below. Find the measures of the angles that surround any vertex point in the tiling.



16. *Developing Proof* $\angle RAC \cong \angle DCA$, $\overline{CD} \cong \overline{AR}$, $\overline{AC} \parallel \overline{DR}$. Is $\overline{AD} \cong \overline{CR}$? Why?



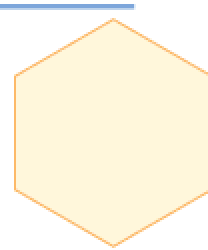
17. *Developing Proof* $\overline{DT} \cong \overline{RT}$, $\overline{DA} \cong \overline{RA}$. Is $\angle D \cong \angle R$? Why?



IMPROVING YOUR VISUAL THINKING SKILLS

Dissecting a Hexagon II

Make six copies of the hexagon at right by tracing it onto your paper. Find six different ways to divide a hexagon into twelve identical parts.

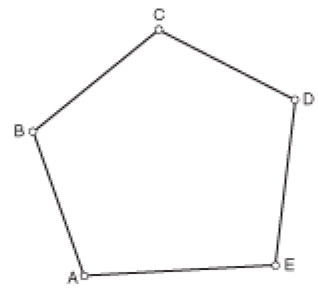


Exploration

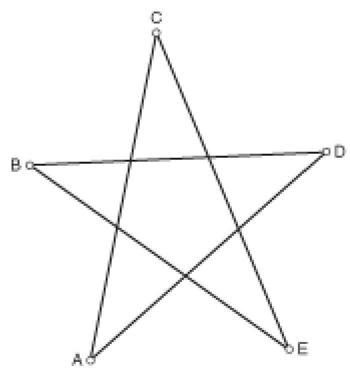
Star Polygons

If you arrange a set of points roughly around a circle or an oval, and then you connect each point to the next with segments, you should get a convex polygon like the one at right. What do you get if you connect every second point with segments? You get a star polygon like the ones shown in the activity below.

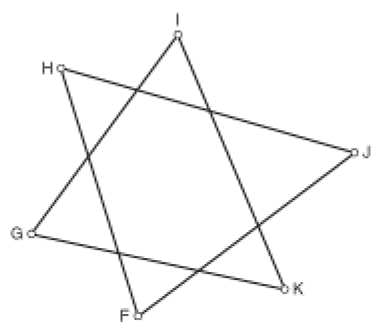
In this activity you'll investigate the angle measure sums of star polygons.



Activity
Exploring Star Polygons



5-pointed star *ABCDE*



6-pointed star *FGHIJK*

Step 1 Draw five points *A* through *E* in a circular path, clockwise.

Step 2 Connect every second point, to get $\overline{AC}$, $\overline{CE}$, $\overline{EB}$, $\overline{BD}$, and $\overline{DA}$.

Step 3 Measure the five angles *A* through *E* at the star points. Use the calculator to find the sum of the angle measures.

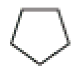
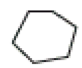
Step 4 Drag each vertex of the star and observe what happens to the angle measures and the calculated sum. Does the sum change? What is the sum?

Step 5 Copy the table on the next page. Use the Polygon Sum Conjecture to complete the first column. Then enter the angle sum for the 5-pointed star.

Step 6 Repeat Steps 1–5 for a 6-pointed star. Enter the angle sum in the table. Complete the column for each *n*-pointed star with every second point connected.

Step 7 What happens if you connect every third point to form a star? What would be the sum of the angle measures in this star? Complete the table column for every third point.

Step 8 Use what you have learned to complete the table. What patterns do you notice? Write the rules for *n*-pointed stars.

	Angle measure sums by how the star points are connected				
Number of star points	Every point	Every 2nd point	Every 3rd point	Every 4th point	Every 5th point
5	 540°				
6	 720°				
7					

Step 9 Let's explore Step 4 a little further. Can you drag the vertices of each star polygon to make it convex? Describe the steps for turning each one into a convex polygon, and then back into a star polygon again, in the fewest steps possible.

Step 10 In Step 9, how did the sum of the angle measure change when a polygon became convex? When did it change?

This blanket by Teresa Archuleta-Sagel is titled *My Blue Vallero Heaven*. Are these star polygons? Why?