Student Web Links @ Keymath.com

LESSON 5.1

Polygon Sum Conjecture

I find that the harder I work, the more luck I seem to have.

THOMAS JEFFERSON

There are many kinds of triangles, but in Chapter 4, you discovered that the sum of their angle measures is always 180°. In this lesson you'll investigate the sum of the angle measures in convex quadrilaterals, pentagons, and other polygons. Then you'll look for a pattern in the sum of the angle measures in *any* polygon.

Investigation

Is There a Polygon Sum Formula?

You will need

- protractor

For this investigation each person in your group should draw a different version of the same polygon. For example, if your group is investigating hexagons, try to think of different ways you could draw a hexagon.

Step 1 Draw the polygon. Carefully measure all the interior angles, then find the sum.

Step 2 Share your results with your group. If you measured carefully, you should all have the same sum! If your answers aren't exactly the same, find the average.

Step 3 Copy the table below. Repeat Steps 1 and 2 with different polygons, or share results with other groups. Complete the table.

Number of sides of polygon	3	4	5	6	7	8	…	n
Sum of measures of angles	180°						…	

You can now make some conjectures.

Quadrilateral Sum Conjecture C-29

The sum of the measures of the four interior angles of any quadrilateral is ?.

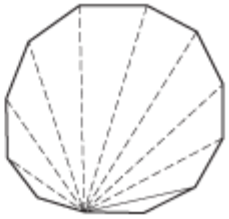
Pentagon Sum Conjecture C-30

The sum of the measures of the five interior angles of any pentagon is ?.

If a polygon has n sides, it is called an ***n*-gon.**

Step 4 Look for a pattern in the completed table. Write a general formula for the sum of the angle measures of a polygon in terms of the number of sides, n.

Step 5 Draw all the diagonals from *one* vertex of your polygon. How many triangles do the diagonals create? How many triangles do the diagonals create in the dodecagon at right? Try some other polygons. How do the number of triangles relate to n?

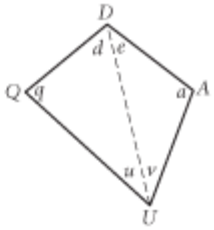


State your observations as a conjecture.

Polygon Sum Conjecture C-31

The sum of the measures of the n interior angles of an n-gon is ?

Developing Proof As a group, write a proof of the Quadrilateral Sum Conjecture using the diagram at right. ■



Does the Polygon Sum Conjecture apply to concave polygons? This question is left for you to explore as a Take Another Look activity.

Keymath.com Practice Your Skills

EXERCISES

You will need

Geometry software for Exercise 19

1. Use the Polygon Sum Conjecture to complete the table.

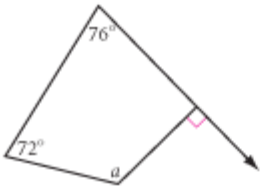
Number of sides of polygon	7	8	9	10	11	20	55	100
Sum of measures of angles								

2. What is the measure of each angle of an equiangular pentagon? An equiangular hexagon? Complete the table. (h)

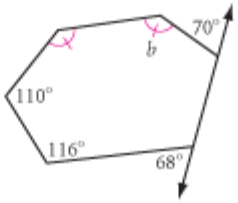
Number of sides of equiangular polygon	5	6	7	8	9	10	12	16	100
Measure of each angle of equiangular polygon									

In Exercises 3–8, use your conjectures to calculate the measure of each lettered angle.

3. a = ?

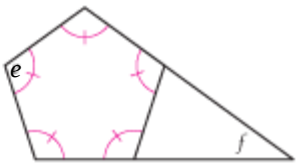


4. b = ?

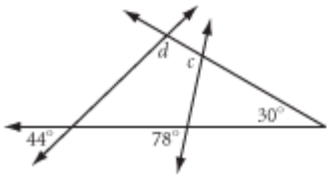


5. e = ?

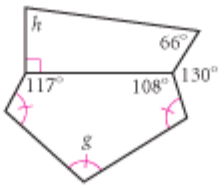
f = ?



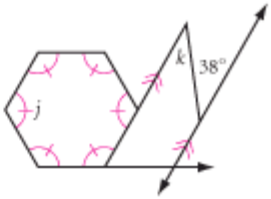
6. c = ? d = ?



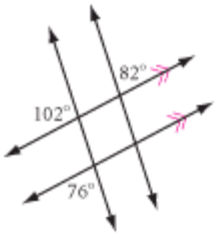
7. g = ? h = ?



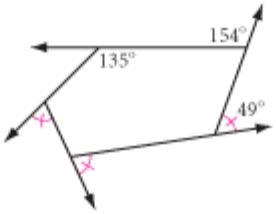
8. j = ? k = ?



9. ***Developing Proof*** What's wrong with this picture?

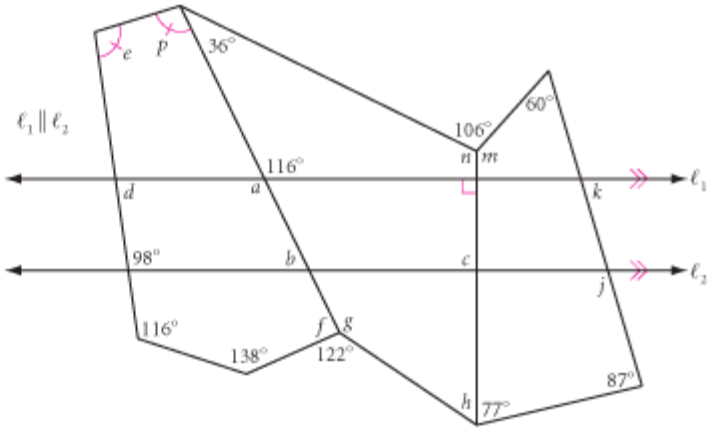


10. ***Developing Proof*** What's wrong with this picture?



11. Three regular polygons meet at point A. How many sides does the largest polygon have?

12. ***Developing Proof*** Trace the figure at right. Calculate each lettered angle measure. Explain how you determined the measures of angles d, e, and f.



13. How many sides does a polygon have if the sum of its angle measures is 2700°?

14. How many sides does an equiangular polygon have if each interior angle measures 156°?

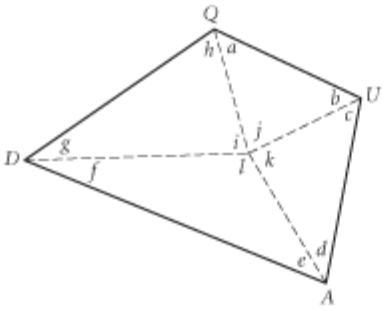
15. Archaeologist Ertha Diggs has uncovered a piece of a ceramic plate. She measures it and finds that each side has the same length and each angle has the same measure.

She conjectures that the original plate was the shape of a regular polygon. She knows that if the original plate was a regular 16-gon, it was probably a ceremonial dish from the third century. If it was a regular 18-gon, it was probably a palace dinner plate from the twelfth century.

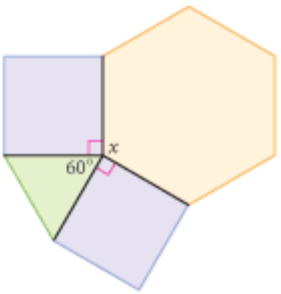
If each angle measures 160°, from what century did the plate likely originate?

16. ***Application*** You need to build a window frame for an octagonal window like this one. To make the frame, you'll cut identical trapezoidal pieces. What are the measures of the angles of the trapezoids? Explain how you found these measures.

17. ***Developing Proof*** Restate your proof of the Quadrilateral Sum Conjecture from the developing proof activity on page 259. Then use this diagram to show another way to prove the same conjecture. (h)



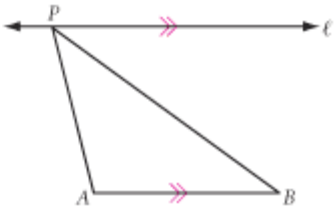
Review



18. This figure is a detail of one vertex of the tiling at the beginning of this lesson. Find the missing angle measure x.

19. ***Technology*** Use geometry software to construct a quadrilateral and locate the midpoints of its four sides. Construct segments connecting the midpoints of opposite sides. Construct the point of intersection of the two segments. Drag a vertex or a side so that the quadrilateral becomes concave. Observe these segments and make a conjecture.

20. Line ℓ is parallel to $\overleftrightarrow{AB}$. As P moves to the right along ℓ, which of these measures will always increase?



A. The distance PA

B. The perimeter of $\triangle ABP$

C. The measure of $\angle APB$

D. The measure of $\angle ABP$

21. Draw a counterexample to show that this statement is false: If a triangle is isosceles, then its base angles are not complementary.

IMPROVING YOUR VISUAL THINKING SKILLS

Net Puzzle

The clear cube shown has the letters *DOT* printed on one face. When a light is shined on that face, the image of *DOT* appears on the opposite face. The image of *DOT* on the opposite face is then painted. Copy the net of the cube and sketch the painted image of the word, *DOT*, on the correct square and in the correct position.