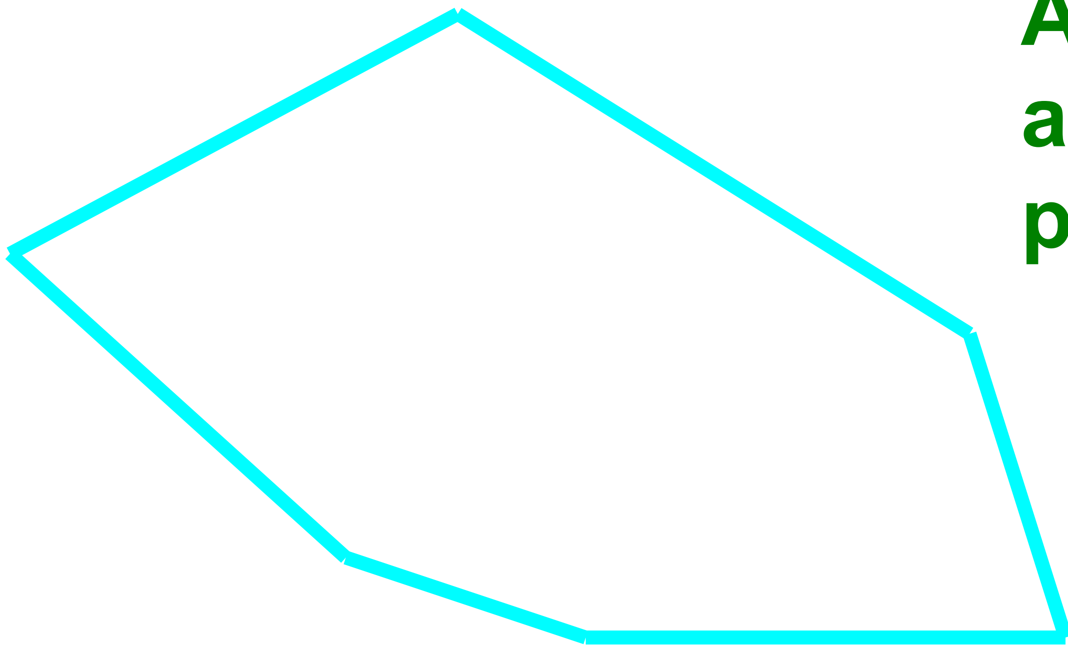
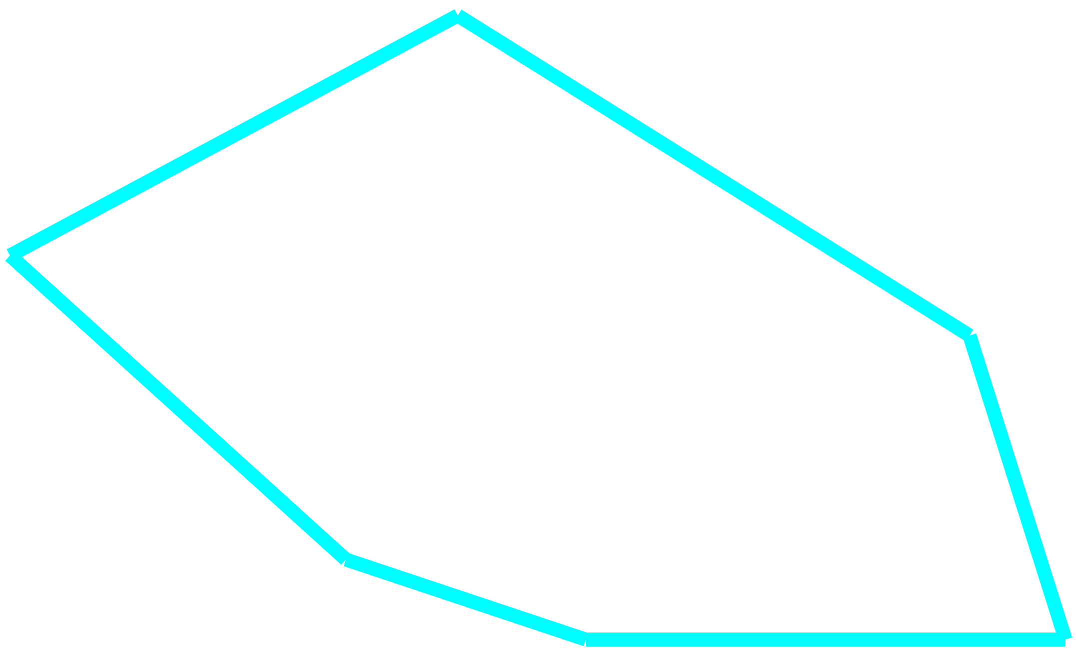
5.2 Polygon Exterior Angles

What is an interior angle?



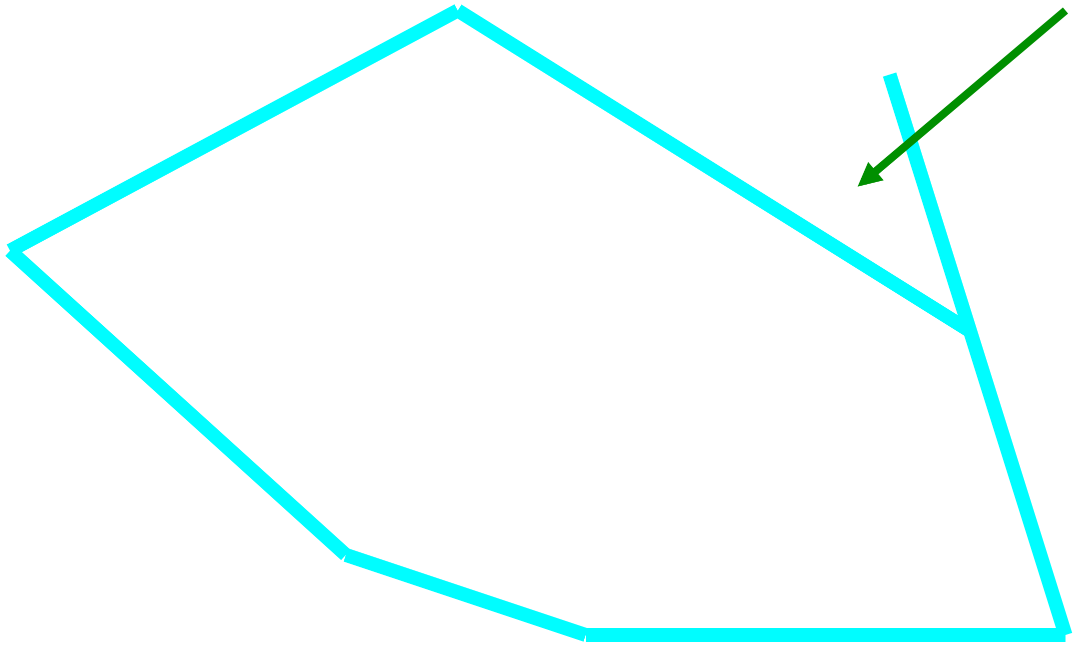
Any of the six angles inside the polygon

What is an exterior angle?



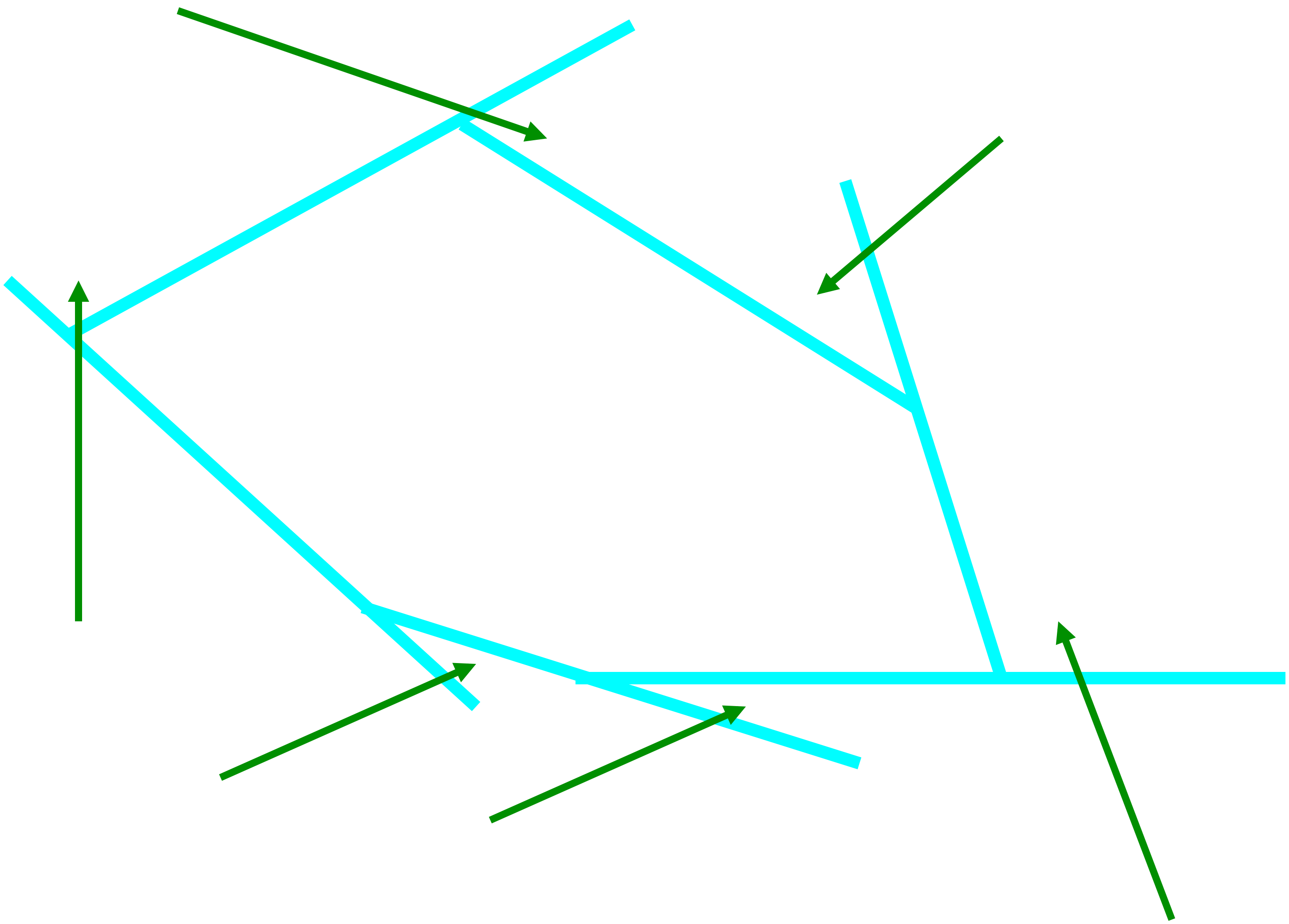
When the sides of the polygon are extended, the angle that is adjacent to the interior angle

What is an exterior angle?

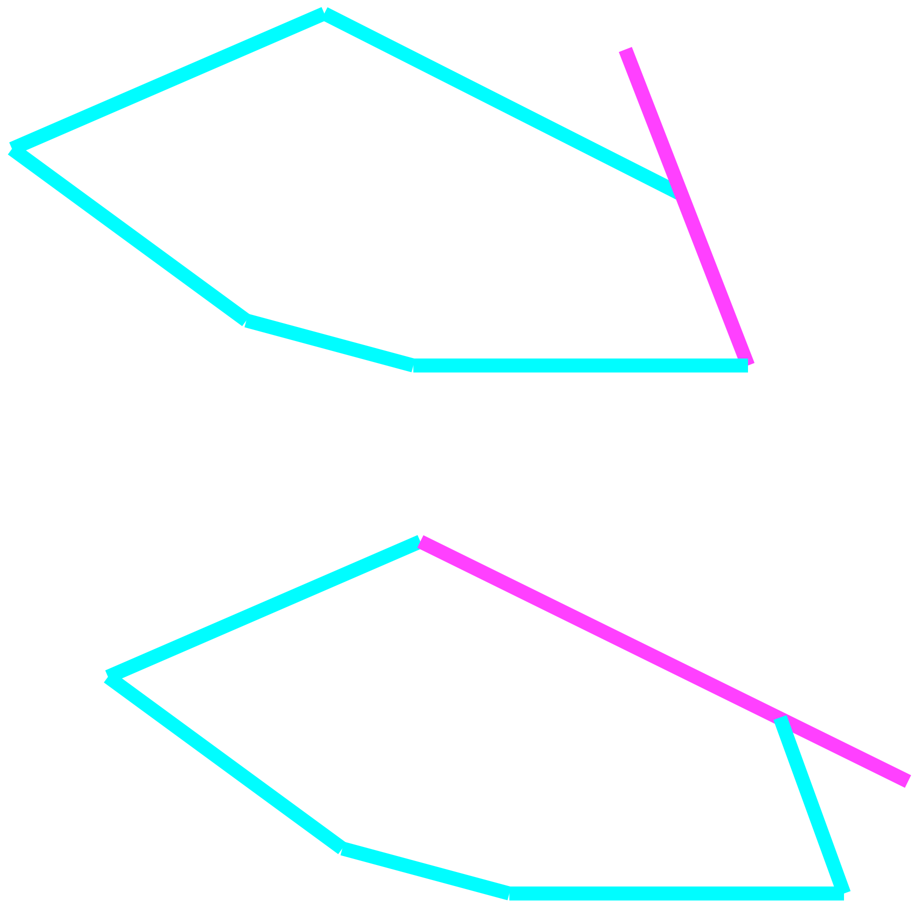


When the sides of the polygon are extended, the angle that is adjacent to the interior angle

Spot the exterior angle

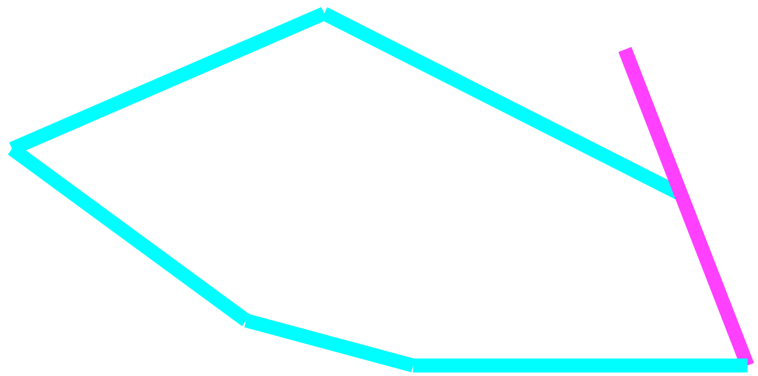


Exterior Angles

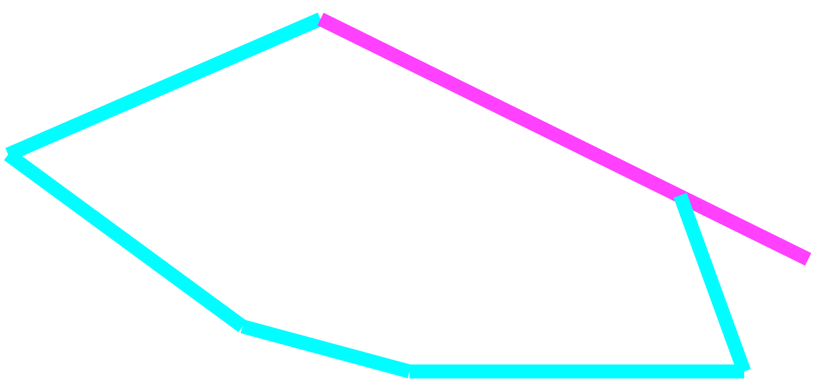


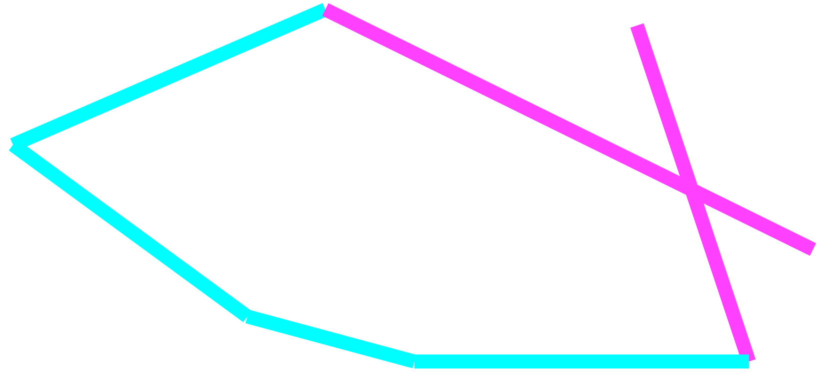
Which side do you extend to create an exterior angle?

Exterior Angles

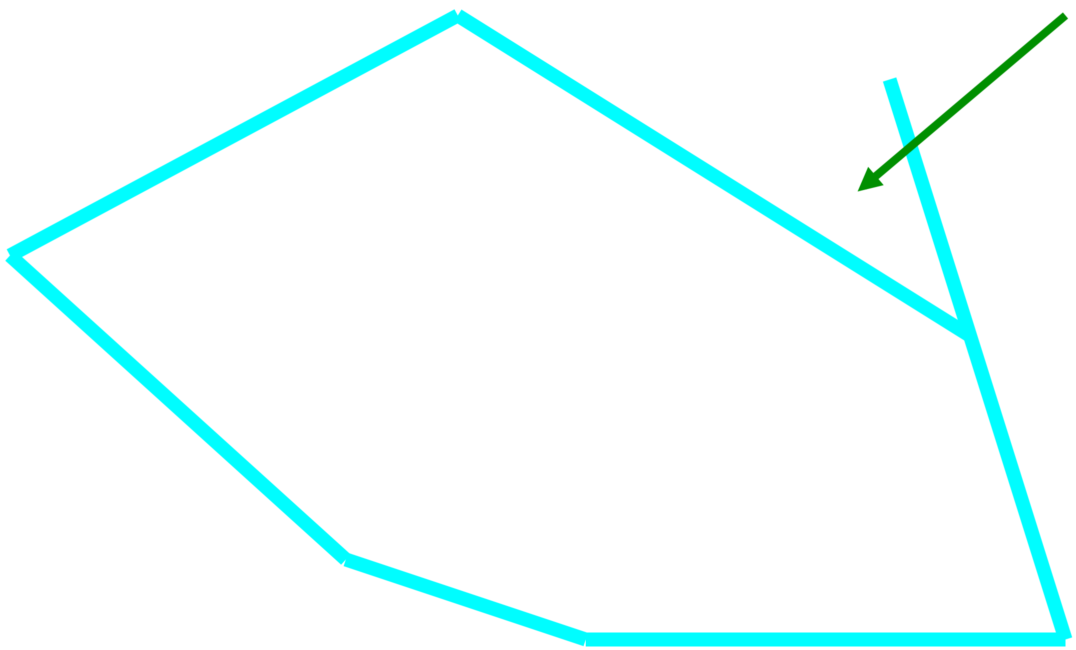


Either one, these two angles are congruent, because they are vertical angles.





Exterior angle



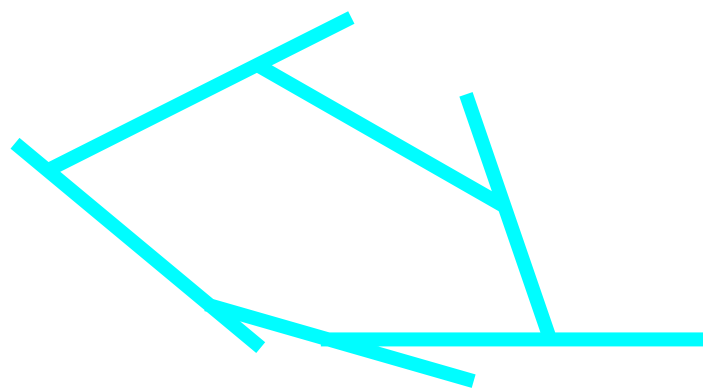
What is the value of all the exterior angles in any polygon?

Exterior Angle Investigation

- Get your supplies
  - Printer Paper
  - Straightedge
  - Protractor

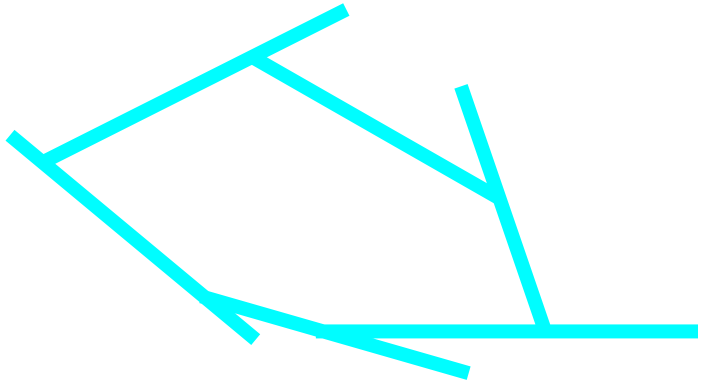
Exterior Angle Investigation

- Draw a large polygon on your paper
  - Pentagon or larger
- Extend each of its sides to form exterior angles



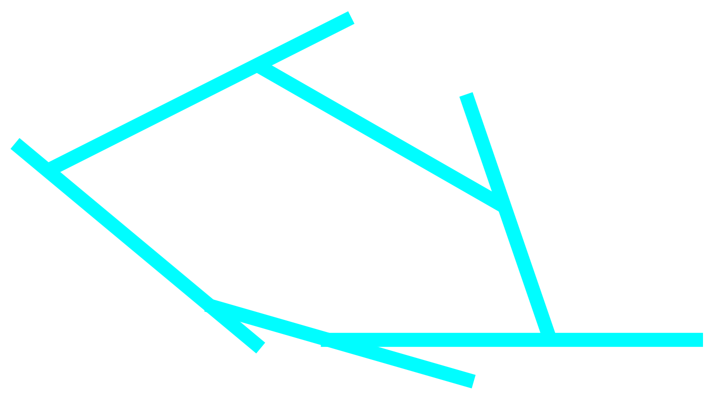
Exterior Angle Investigation

- Measure all of the interior angles except one. Write the value of the interior angle on the angle.
- Use the polygon sum conjecture to calculate the measure of the last angle before you measure it.



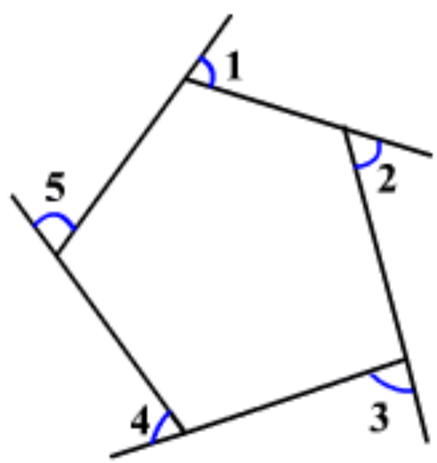
Exterior Angle Investigation

- Use the linear pair conjecture to calculate the value of each exterior angle.
- Write down the value on the angle
- Add up all the exterior measures



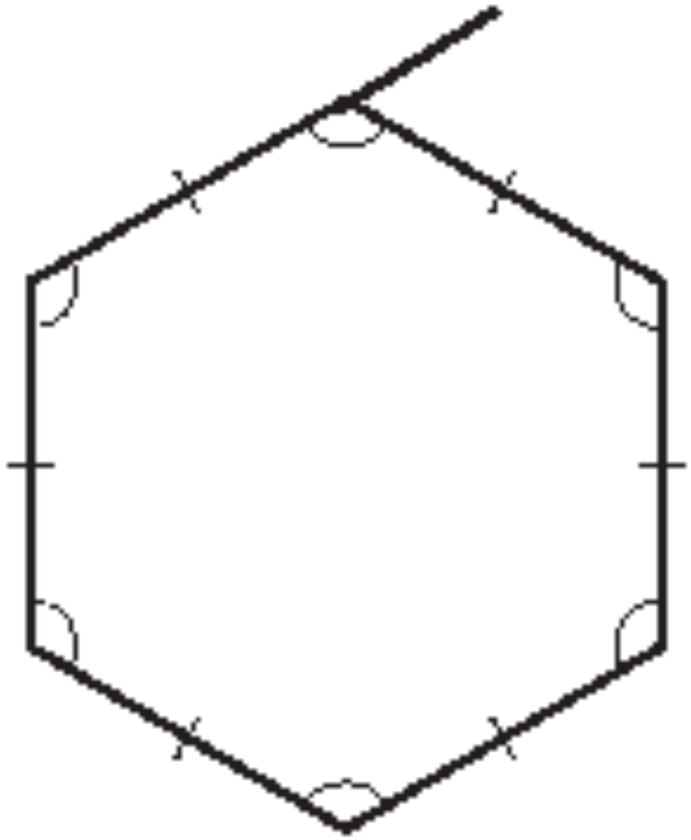
Exterior Angle Theorem

- The sum of the measures of the exterior angles of a convex polygon, (one angle at each vertex), is 360°



$$m\angle 1 + m\angle 2 + m\angle 3 + m\angle 4 + m\angle 5 = 360°$$

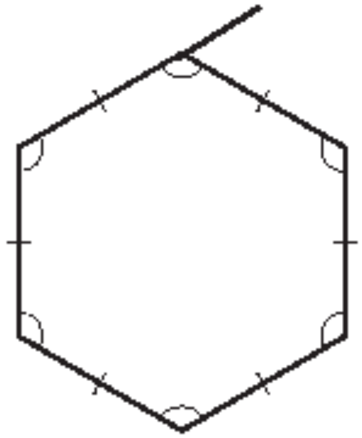
Exterior angle



What is the measure of an exterior angle in a regular polygon?

Exterior Angle Theorem

- The measure of each exterior angle of a regular n-gon is 360°/n



Equiangular Polygon Conjecture

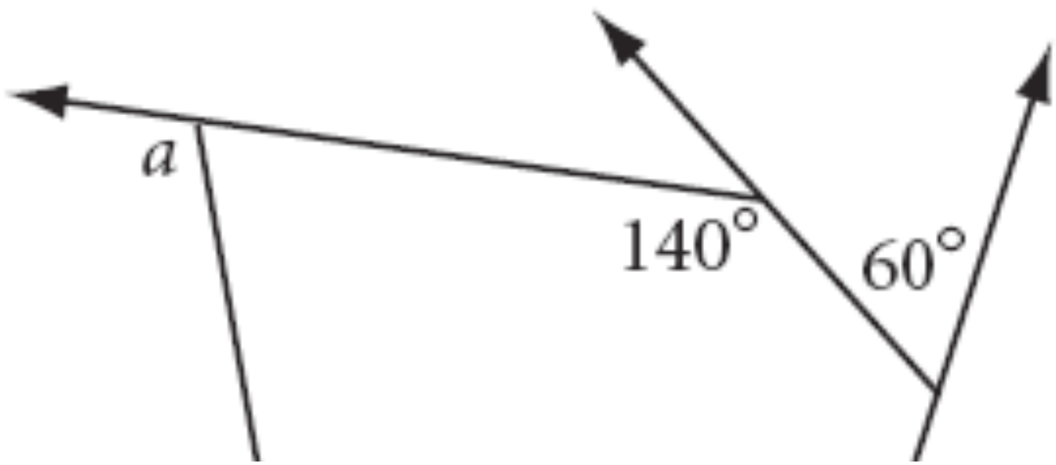
- You can find the measure of each interior angle of an equiangular n-gon by using either of these formulas:

$$180° - 360°/n$$

$$(180°(n-2))/n$$

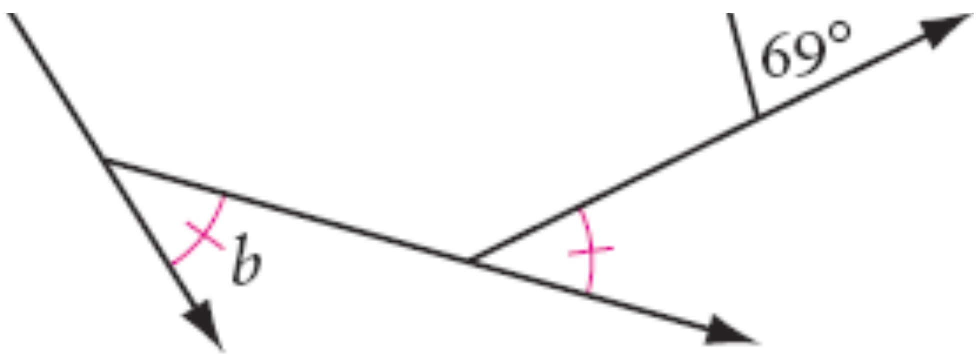
Practice Problems

calculate the measure of each lettered angle.



$a = 108°$. Use the Exterior Angle Sum Conjecture. The figure shows a set of exterior angles (one at each vertex) for a pentagon. The measure of the unmarked exterior angle is $180° - 140° = 40°$ (Linear Pair Conjecture). Therefore, $a + 68° + 84° + 60° + 40° = 360°$, so $a = 108°$.

c: $b = 45\frac{1}{3}°$. Use the Exterior Angle Sum Conjecture. The measure of the unmarked exterior angle is $180° - 68° = 112°$ (Linear Pair Conjecture). There are three exterior angles with measure b. Therefore, $3b + 69° + 43° + 112° = 360°$, so $3b = 136°$, and $b = 45\frac{1}{3}°$.



calculate the measure of each lettered angle.

$c = 51\frac{3}{7}°$, $d = 115\frac{5}{7}°$. By the Equiangular Polygon Conjecture, the measure of an interior angle of an equiangular heptagon is $\frac{5 \cdot 180°}{7} = 128\frac{4}{7}°$. Then $c = 180° - 128\frac{4}{7}° = 51\frac{3}{7}°$ (Linear Pair Conjecture). Now look at the common vertex of the heptagon and the two quadrilaterals. The sum of the angle measures at this vertex is 360°, so $2d + 128\frac{4}{7}° = 360°$. Therefore, $2d = 231\frac{3}{7}°$, and $d = 115\frac{5}{7}°$.

calculate the measure of each lettered angle.

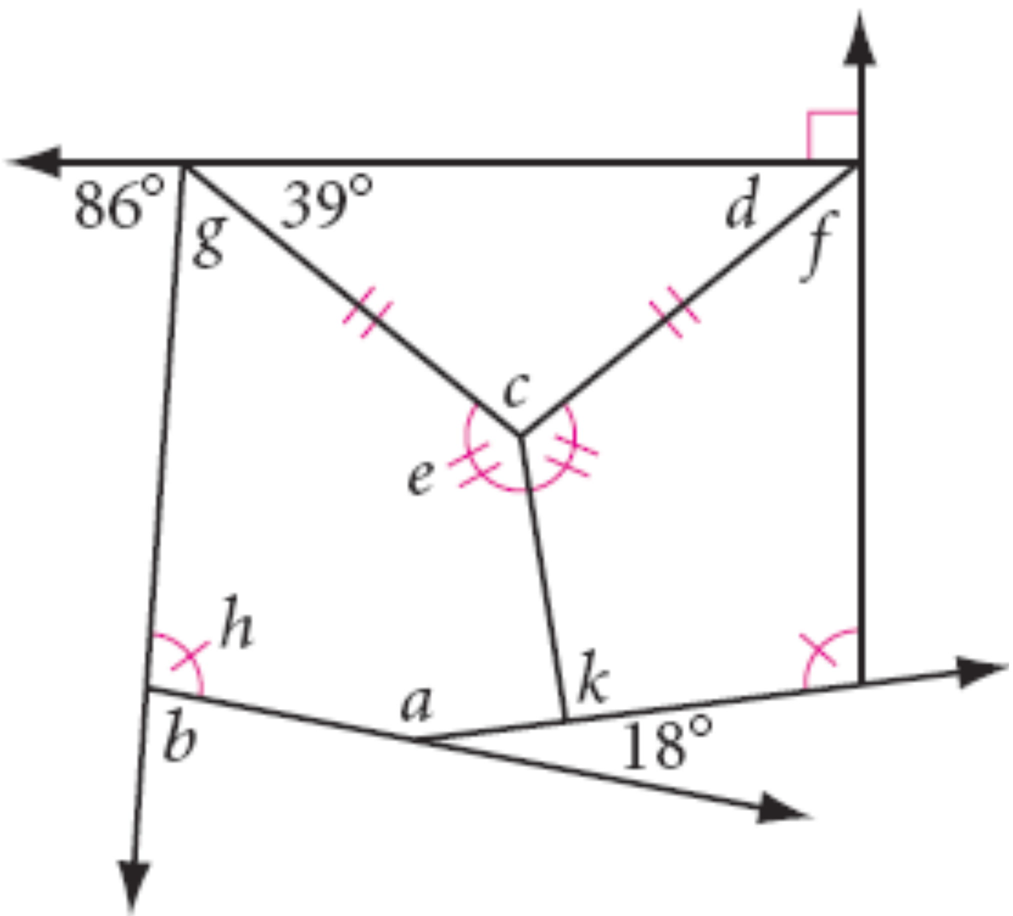
$e = 72°$, $f = 45°$, $g = 117°$, and $h = 126°$. Use the Equiangular Polygon Conjecture. In the regular pentagon, the measure of each interior angle is $\frac{3 \cdot 180°}{5} = 108°$, so $e = 180° - 108° = 72°$. In

the regular octagon, the measure of each interior angle is $\frac{6 \cdot 180°}{8} = 135°$, so $f = 180° - 135° = 45°$. g is the measure of one of three angles whose sum is 360°. Because one of these angles is an interior angle of the pentagon and another is an interior angle of the octagon, $g + 108° + 135° = 360°$, so $g = 117°$. Finally, $h + e + f + g = 360°$ (Quadrilateral Sum Conjecture), so $h + 234° = 360°$, and $h = 126°$.

calculate the measure of each lettered angle.

$a = 30°$, $b = 30°$, $c = 106°$, and $d = 136°$. First, $a + 56° + 94° = 180°$ (Triangle Sum Conjecture), so $a = 30°$. Next, $b = a$ (AIA Conjecture), so $b = 30°$. From the triangle on the left, $b + c + 44° = 180°$, so $c = 106°$. Finally, look at the quadrilateral that contains angles with measures 56°, 94°, and d, as well as an unmarked angle. The measure of the unmarked angle is $180° - c = 74°$ (Linear Pair Conjecture), so $d + 94° + 56° + 74° = 360°$ (Quadrilateral Sum Conjecture), and $d = 136°$.

calculate the measure of



$a = 162°$, $b = 83°$, $c = 102°$, $d = 39°$, $e = 129°$, $f = 51°$, $g = 55°$, $h = 97°$, and $k = 83°$.

$a = 180° - 18° = 162°$. To find b, look at the exterior angles of the large pentagon (which is subdivided into a triangle, a quadrilateral, and a pentagon). The unmarked exterior angle at the lower right forms a linear pair with an angle that is marked as congruent to the angle with measure h, so the measure of this exterior angle is b. The Exterior Angle Sum Conjecture says that the sum of the measures of a set of exterior angles (one at each vertex) of any polygon is 360°. Therefore, $86° + b + 18° + b + 90° = 360°$, or $2b + 194° = 360°$, and $b = 83°$. Next look at the isosceles triangle. Here, $d = 39°$ (Isosceles Triangle Conjecture), and $2 \cdot 39° + c = 180°$ (Triangle Sum Conjecture), so $c = 102°$. Next look at the vertex of the triangle with measure c. Here, $2e + c = 360°$, so $2e = 258°$ and $e = 129°$. Now look at the upper-right corner of the figure. Here, $d + f + 90° = 180°$, so $f = 51°$. Next look at the upper-left corner of the figure. Here, $86° + g + 39° = 180°$, so $g = 55°$. Now look at the lower-left corner. $h + b = 180°$, so $h = 97°$. Finally, look at the quadrilateral. The angle in the lower-right corner of the quadrilateral is congruent to the angle with measure h, so its measure is also 97°. By the Quadrilateral Sum Conjecture, $k + 97° + f + 129° = 360°$, or $k + 277° = 360°$, so $k = 83°$.

Homework

- Worksheet 5-1
- Worksheet 5-2