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LESSON 3.1

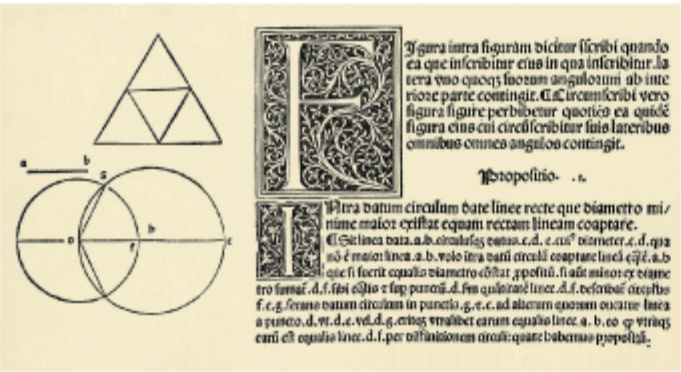
# Duplicating Segments and Angles

*It is only the first step that is difficult.*

MARIE DE VICHY-CHAMROD

The compass, like the straightedge, has been a useful geometry tool for thousands of years. The ancient Egyptians used the compass to mark off distances. During the Golden Age of Greece, Greek mathematicians made a game of geometric constructions. In his work *Elements,* Euclid (325–265 B.C.E.) established the basic rules for constructions using only a compass and a straightedge. In this course you will learn how to construct geometric figures using these tools as well as patty paper.

Constructions with patty paper are a variation on the ancient Greek game of geometric constructions. Almost all the figures that can be constructed with a compass and a straightedge can also be constructed using a straightedge and patty paper, waxed paper, or tracing paper. If you have access to a computer with a geometry software program, you can do constructions electronically.



In the previous chapters, you drew and sketched many figures. In this chapter, however, you'll **construct** geometric figures. The words *sketch, draw,* and *construct* have specific meanings in geometry.

## Mathematics CONNECTION

Euclidean geometry is the study of geometry based on the assumptions of Euclid (325–265 B.C.E.). Euclid established the basic rules for constructions using only a compass and a straightedge. In his work *Elements,* Euclid proposed definitions and constructions about points, lines, angles, surfaces, and solids. He also explained why the constructions were correct with deductive reasoning.

A page from a book on Euclid, above, shows some of his constructions and a translation of his explanations from Greek into Latin.

When you *sketch* an equilateral triangle, you may make a freehand sketch of a triangle that looks equilateral. You don't need to use any geometry tools.

When you *draw* an equilateral triangle, you should draw it carefully and accurately, using your geometry tools. You may use a protractor to measure angles and a ruler to measure the sides to make sure they are equal in measure.

When you *construct* an equilateral triangle with a compass and straightedge, you don't rely on measurements from a protractor or ruler. You must use only a compass and a straightedge. This method of construction guarantees that your triangle is equilateral.

When you *construct* an equilateral triangle with patty paper and straightedge, you fold the paper and trace equal segments. You may use a straightedge to draw a segment, but you may not use a compass or any measuring tools.

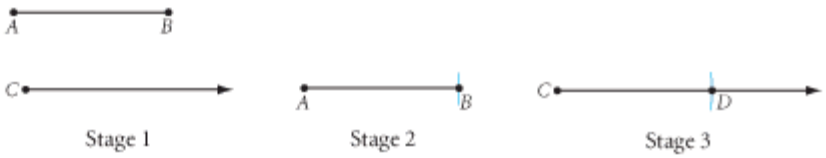
When you sketch or draw, use the special marks that indicate right angles, parallel segments, and congruent segments and angles.

By tradition, neither a ruler nor a protractor is ever used to perform geometric constructions, because no matter how precise we try to be, measurement always involves some amount of inaccuracy. Rulers and protractors are measuring tools, not construction tools. You may use a ruler as a straightedge in constructions, provided you do not use its marks for measuring. In the next two investigations you will discover how to duplicate a line segment and an angle using only your compass and straightedge, or using only patty paper and a straightedge. By **duplicate,** we mean to copy using construction tools.

## Investigation 1
## Duplicating a Segment

**You will need**

- a compass
- a straightedge
- a ruler
- patty paper




**Step 1** The complete construction for copying a segment, $\overline{AB}$, is shown above. Describe each stage of the process.

**Step 2** Use a ruler to measure $\overline{AB}$ and $\overline{CD}$. How do the two segments compare?

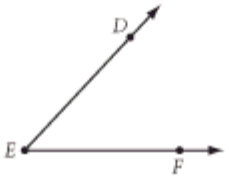
**Step 3** Describe how to duplicate a segment using patty paper instead of a compass.

Using only a compass and a straightedge, how would you duplicate an angle? In other words, how would you construct an angle that is congruent to a given angle? You may not use your protractor, because a protractor is a measuring tool, not a construction tool.

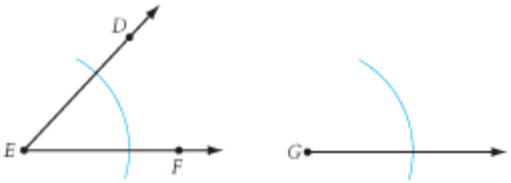
## Investigation 2
## Duplicating an Angle

**You will need**

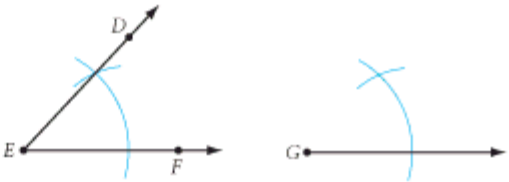
- a compass
- a straightedge




**Step 1** The first two stages for copying $\angle DEF$ are shown below. Describe each stage of the process.




Stage 1




Stage 2

**Step 2** What will be the final stage of the construction?

**Step 3** Use a protractor to measure $\angle DEF$ and $\angle G$. What can you state about these angles?

**Step 4** Describe how to duplicate an angle using patty paper instead of a compass.

You've just discovered how to duplicate segments and angles using a straightedge and compass or patty paper. These are the basic constructions. You will use combinations of these to do many other constructions. You may be surprised that you can construct figures more precisely *without* using a ruler or protractor!

Called *vintas*, these canoes with brightly patterned sails are used for fishing in Zamboanga, Philippines. What angles and segments are duplicated in this photo?

## EXERCISES

Keymath.com Practice Your Skills

You will need

- Construction tools for Exercises 1–10
- Geometry software for Exercise 11

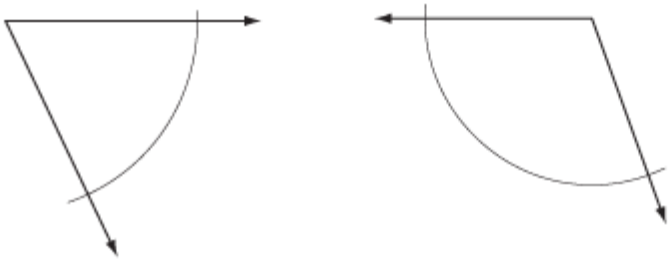
***Construction*** Now that you can duplicate line segments and angles using construction tools, do the constructions in Exercises 1–10. You will duplicate polygons in Exercises 7 and 10.

**1.** Using only a compass and a straightedge, duplicate the three line segments shown below. Label them as they're labeled in the figures.


**2.** Use the segments from Exercise 1 to construct a line segment with length $AB + CD$.

**3.** Use the segments from Exercise 1 to construct a line segment with length $2AB + 2EF - CD$.

**4.** Use a compass and a straightedge to duplicate each angle. There's an arc in each angle to help you.



**5.** Draw an obtuse angle. Label it *LGE*, then duplicate it.

**6.** Draw two acute angles on your paper. Construct a third angle with a measure equal to the sum of the measures of the first two angles. Remember, you cannot use a protractor—use a compass and a straightedge only.

**7.** Draw a large acute triangle on the top half of your paper. Duplicate it on the bottom half, using your compass and straightedge. Do not erase your construction marks, so others can see your method.

**8.** Construct an equilateral triangle. Each side should be the length of this segment.



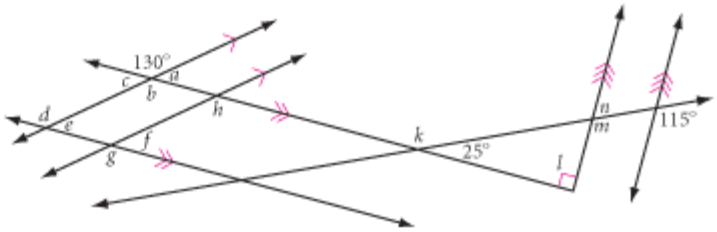
**9.** Repeat Exercises 7 and 8 using constructions with patty paper.

**10.** Draw quadrilateral *QUAD*. Duplicate it, using your compass and straightedge. Label the construction *COPY* so that $QUAD \cong COPY$.

**11.** ***Technology*** Use geometry software to construct an equilateral triangle. Drag each vertex to make sure it remains equilateral.

## Review

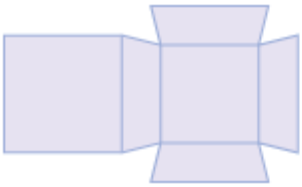
**12.** Copy the diagram at right. Use the Vertical Angles Conjecture and the Parallel Lines Conjecture to calculate the measure of each angle.



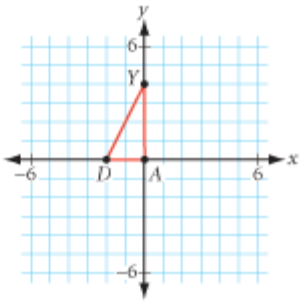

**13.** Hyacinth is standing on the curb waiting to cross 24th Street. A half block to her left is Avenue J, and Avenue K is a half block to her right. Numbered streets run parallel to one another and are all perpendicular to lettered avenues. If Avenue P is the northernmost avenue, which direction (north, south, east, or west) is she facing?

**14.** Write a new definition for an isosceles triangle, based on the triangle's reflectional symmetry. Does your definition apply to equilateral triangles? Explain. (h)

**15.** Sketch the three-dimensional figure formed by folding this net into a solid.



**16.** Draw △*DAY* after it is rotated 90° clockwise about the origin. Label the coordinates of the vertices.




**17.** Use your ruler to draw a triangle with side lengths 8 cm, 10 cm, and 11 cm. Explain your method. Can you draw a second triangle with the same three side lengths that is not congruent to the first? (h)

## IMPROVING YOUR ALGEBRA SKILLS


### *Pyramid Puzzle II*

Place four different numbers in the bubbles at the vertices of each pyramid so that the two numbers at the ends of each edge add to the number on that edge.