

*Discovery consists of looking at the same thing as everyone else and thinking something different.*

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## Angle Relationships

Now that you've had experience with inductive reasoning, let's use it to start discovering geometric relationships. This investigation is the first of many investigations you will do using your geometry tools.



Create an investigation section in your notebook. Include a title and illustration for each investigation and write a statement summarizing the results of each one.

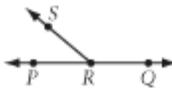


### Investigation 1

#### The Linear Pair Conjecture

##### You will need

- a protractor



- Step 1 On a sheet of paper, draw  $\overline{PQ}$  and place a point  $R$  between  $P$  and  $Q$ . Choose another point  $S$  not on  $\overline{PQ}$  and draw  $\overline{RS}$ . You have just created a linear pair of angles. Place the “zero edge” of your protractor along  $\overline{PQ}$ . What do you notice about the sum of the measures of the linear pair of angles?
- Step 2 Compare your results with those of your group. Does everyone make the same observation? Complete the statement.

#### Linear Pair Conjecture

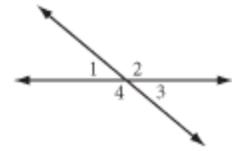
C-1

If two angles form a linear pair, then ?.



The important conjectures have been given a name and a number. Start a list of them in your notebook. The Linear Pair Conjecture (C-1) and the Vertical Angles Conjecture (C-2) should be the first entries on your list. Make a sketch for each conjecture.

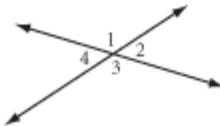
In the previous investigation you discovered the relationship between a linear pair of angles, such as  $\angle 1$  and  $\angle 2$  in the diagram at right. You will discover the relationship between vertical angles, such as  $\angle 1$  and  $\angle 3$ , in the next investigation.



## Investigation 2 Vertical Angles Conjecture

### You will need

- a straightedge
- patty paper



- Step 1** Draw two intersecting lines onto patty paper or tracing paper. Label the angles as shown. Which angles are vertical angles?
- Step 2** Fold the paper so that the vertical angles lie over each other. What do you notice about their measures?
- Step 3** Fold the paper so that the other pair of vertical angles lie over each other. What do you notice about their measures?
- Step 4** Compare your results with the results of others. Complete the statement.



### Vertical Angles Conjecture

C-2

If two angles are vertical angles, then     ?

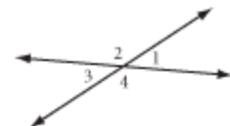


**Developing Proof** You used inductive reasoning to discover both the Linear Pair Conjecture and the Vertical Angles Conjecture. Are they related? If you accept the Linear Pair Conjecture as true, can you use deductive reasoning to show that the Vertical Angles Conjecture must be true?

Read the example below. Without turning the page, write a deductive argument with your group. Remember the reasoning strategy of representing a situation algebraically. Another strategy is to apply previous conjectures and definitions to a new situation. Then compare your solution to the one on the next page. ■

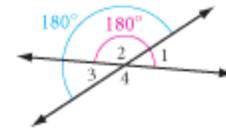
### EXAMPLE

Use the Linear Pair Conjecture and the diagram at right to write a deductive argument explaining why  $\angle 1$  must be congruent to  $\angle 3$ .



► **Solution**

You can see from the diagram that the sum of the measures of angles 1 and 2 is equal to the sum of the measures of angles 2 and 3 because they are both linear pairs. Because angle 2 is the same in both sums, angle 1 must equal angle 3. To write a deductive argument, go through this logic one step at a time.



**Deductive Argument**

For any linear pair of angles, their measures add up to  $180^\circ$ .

$$m\angle 1 + m\angle 2 = 180^\circ$$

$$m\angle 2 + m\angle 3 = 180^\circ$$

Since both expressions on the left equal  $180^\circ$ , they equal each other.

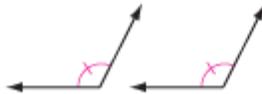
$$m\angle 1 + m\angle 2 = m\angle 2 + m\angle 3$$

Subtract  $m\angle 2$  from both sides of the equation.

$$m\angle 1 = m\angle 3$$

Vertical angles 1 and 3 have equal measures, so they are congruent. ■

You discovered the Vertical Angles Conjecture: If two angles are vertical angles, then they are congruent. Does that also mean that all congruent angles are vertical angles? The **converse** of an “if-then” statement switches the “if” and “then” parts. The converse of the Vertical Angles Conjecture may be stated: If two angles are congruent, then they are vertical angles. Is this converse statement true? Remember that if you can find even one counterexample, like the diagram below, then the statement is false.



Therefore, the converse of the Vertical Angles Conjecture is false.

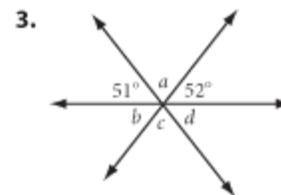
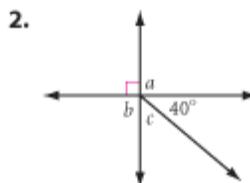
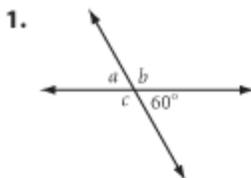


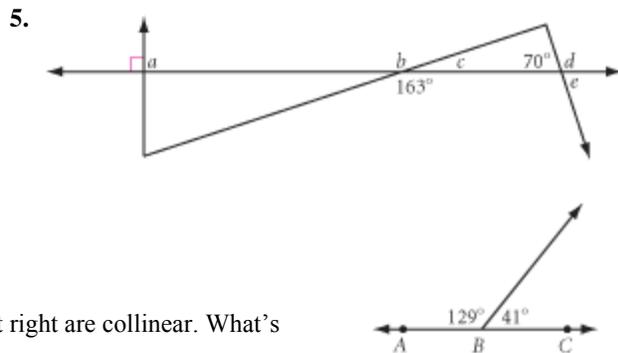
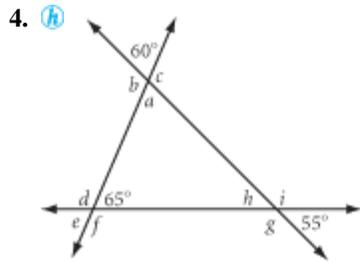
**EXERCISES**

**You will need**



Without using a protractor, but with the aid of your two new conjectures, find the measure of each lettered angle in Exercises 1–5. Copy the diagrams so that you can write on them. List your answers in alphabetical order.



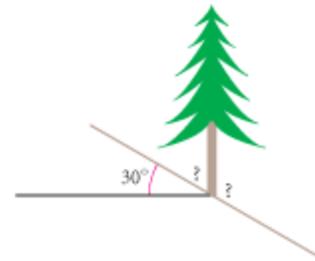


6. **Developing Proof** Points  $A$ ,  $B$ , and  $C$  at right are collinear. What's wrong with this picture?

7. Yoshi is building a cold frame for his plants. He wants to cut two wood strips so that they'll fit together to make a right-angled corner. At what angle should he cut ends of the strips?



8. A tree on a  $30^\circ$  slope grows straight up. What are the measures of the greatest and smallest angles the tree makes with the hill? Explain.



9. You discovered that if a pair of angles is a linear pair, then the angles are supplementary. Does that mean that all supplementary angles form a linear pair of angles? Is the converse true? If not, sketch a counterexample.

10. If two congruent angles are supplementary, what must be true of the two angles? Make a sketch, then complete the following conjecture: If two angles are both congruent and supplementary, then  $\underline{\quad ? \quad}$ .

11. **Developing Proof** Using algebra, write a paragraph proof that explains why the conjecture from Exercise 10 is true.

12. **Technology** Use geometry software to construct two intersecting lines. Measure a pair of vertical angles. Use the software to calculate the ratio of their measures. What is the ratio? Drag one of the lines. Does the ratio ever change? Does this demonstration convince you that the Vertical Angles Conjecture is true? Does it explain why it is true?

## Review

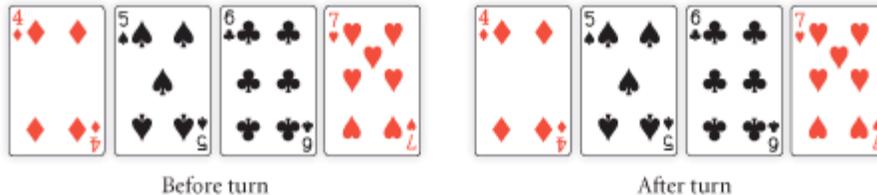
For Exercises 13–17, sketch, label, and mark the figure.

13. Scalene obtuse triangle  $PAT$  with  $PA = 3$  cm,  $AT = 5$  cm, and  $\angle A$  an obtuse angle

14. A quadrilateral that has rotational symmetry, but not reflectional symmetry

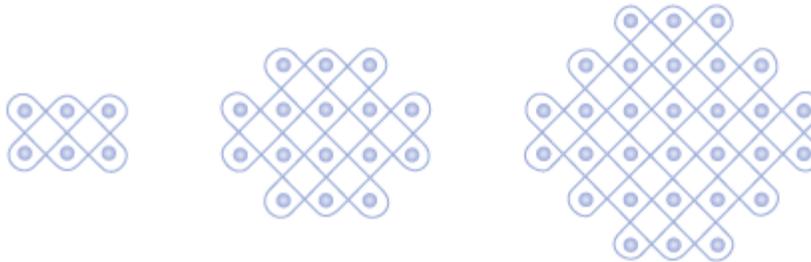
15. A circle with center at  $O$  and radii  $\overline{OA}$  and  $\overline{OT}$  creating a minor arc  $\widehat{AT}$

16. A pyramid with an octagonal base
17. A 3-by-4-by-6-inch rectangular solid rests on its smallest face. Draw lines on the three visible faces to show how you can divide it into 72 identical smaller cubes.
18. Miriam the Magnificent placed four cards face up (the first four cards shown below). Blindfolded, she asked someone from her audience to come up to the stage and turn one card  $180^\circ$ .

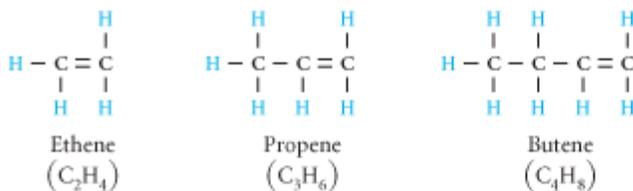


Miriam removed her blindfold and claimed she was able to determine which card was turned  $180^\circ$ . What is her trick? Can you figure out which card was turned? Explain.

19. If a pizza is cut into 16 congruent pieces, how many degrees are in each angle at the center of the pizza?
20. Paulus Gerdes, a mathematician from Mozambique, uses traditional *lusona* patterns from Angola to practice inductive thinking. Shown below are three *sona* designs. Sketch the fourth *sona* design, assuming the pattern continues.



21. Hydrocarbon molecules in which all the bonds between the carbon atoms are single bonds except one double bond are called *alkenes*. The first three alkenes are modeled below.



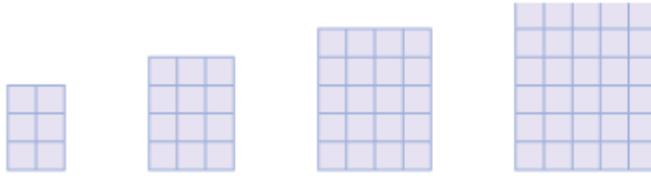
Sketch the alkene with eight carbons in the chain. What is the general rule for alkenes ( $\text{C}_n\text{H}_?$ )? In other words, if there are  $n$  carbon atoms (C), how many hydrogen atoms (H) are in the alkene?

### Science

#### CONNECTION

Organic chemistry is the study of carbon compounds and their reactions. Drugs, vitamins, synthetic fibers, and food all contain organic molecules. To learn about new advances in organic chemistry, go to [www.keymath.com/DG](http://www.keymath.com/DG).

22. If the pattern of rectangles continues, what is the rule for the perimeter of the  $n$ th rectangle, and what is the perimeter of the 200th rectangle? What is the rule for the number of 1-by-1 squares in the  $n$ th rectangle, and how many 1-by-1 squares are in the 200th rectangle?



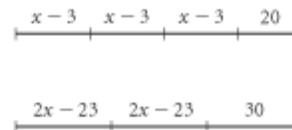
Rectangle	1	2	3	4	5	6	...	$n$	...	200
Perimeter of rectangle	10	14	18				...		...	
Number of squares	6	12	20				...		...	

23. The twelfth-grade class of 80 students is assembled in a large circle on the football field at halftime. Each student is connected by a string to each of the other class members. How many pieces of string are necessary to connect each student to all the others? 
24. If you draw 80 lines on a piece of paper so that no 2 lines are parallel to each other and no 3 lines pass through the same point, how many intersections will there be? 
25. If there are 20 couples at a party, how many different handshakes can there be between pairs of people? Assume that the two people in each couple do not shake hands with each other. 
26. If a polygon has 24 sides, how many diagonals are there from each vertex? How many diagonals are there in all?
27. If a polygon has a total of 560 diagonals, how many vertices does it have? 
28. A midpoint divides a segment into two congruent segments. Point  $M$  divides segment  $\overline{AY}$  into two congruent segments  $\overline{AM}$  and  $\overline{MY}$ . What conclusion can you make? What type of reasoning did you use?

## IMPROVING YOUR ALGEBRA SKILLS

### Number Line Diagrams

1. The two segments at right have the same length. Translate the number line diagram into an equation, then solve for the variable  $x$ .



2. Translate this equation into a number line diagram.  
 $2(x + 3) + 14 = 3(x - 4) + 11$

