Student Web Links @ Keymath.com

LESSON 1.8

Space Geometry

When curiosity turns to serious matters, it's called research.

MARIE VON EBNER-ESCHENBACH

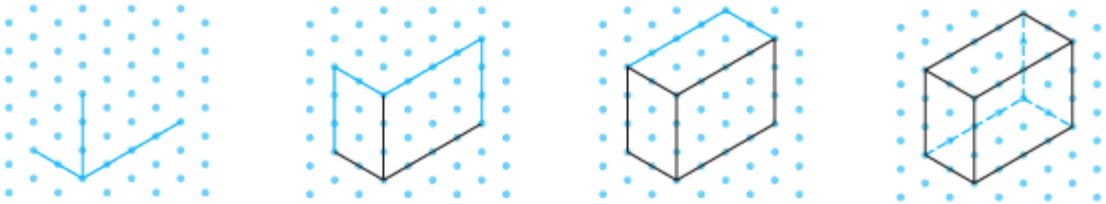
Lesson 1.1 introduced you to point, line, and plane. Throughout this chapter you have used these terms to define a wide range of other geometric figures, from rays to polygons. You did most of your work on a single flat surface, a single plane. Some problems, however, required you to step out of a single plane to visualize geometry in space. In this lesson you will learn more about space geometry, or solid geometry.

Space is the set of all points. Unlike one-dimensional lines and two-dimensional planes, space cannot be contained in a flat surface. Space is three-dimensional, or "3-D."

In an "edge view," you see the front edge of a building as a vertical line, and the other edges as diagonal lines. Isometric dot paper helps you draw these lines, as you can see in the steps below.

Let's practice the visual thinking skill of presenting three-dimensional (3-D) objects in two-dimensional (2-D) drawings.

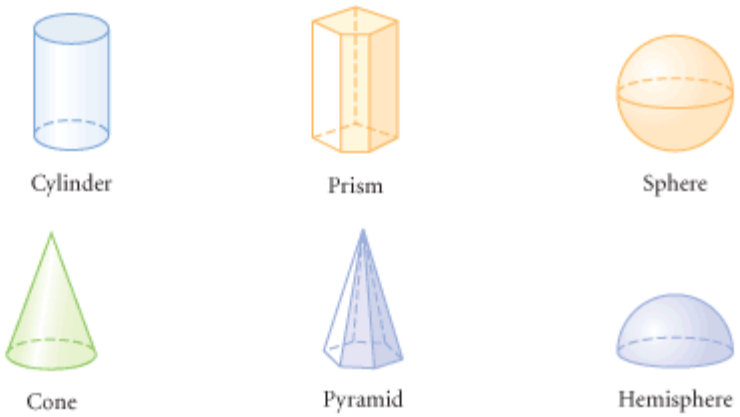
The geometric solid you are probably most familiar with is a box, or rectangular prism. Below are steps for making a two-dimensional drawing of a rectangular prism. This type of drawing is called an **isometric drawing.** It shows three sides of an object in one view (an edge view). This method works best with isometric dot paper. After practicing, you will be able to draw the box without the aid of the dot grid.



Step 1 Step 2 Step 3 Step 4

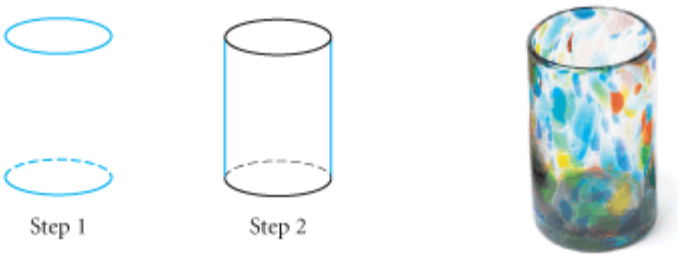
Use dashed lines for edges that you couldn't see if the object were solid.

The three-dimensional objects you will study include the six types of geometric solids shown below.

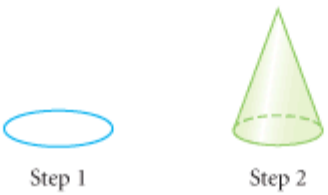


The shapes of these solids are probably already familiar to you even if you are not familiar with their proper names. The ability to draw these geometric solids is an important visual thinking skill. Here are some drawing tips. Remember to use dashes for the hidden lines.

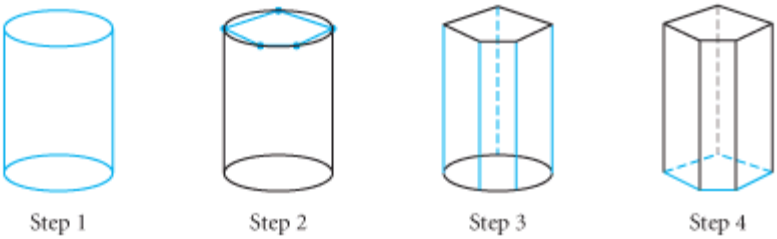
Cylinder



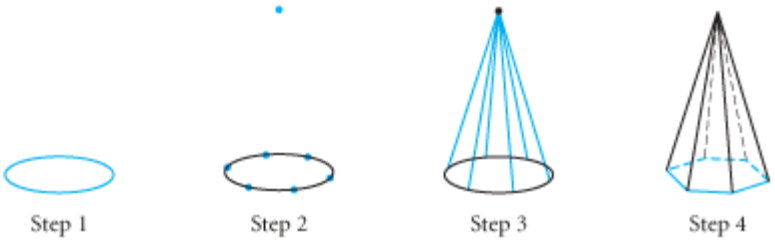
Cone



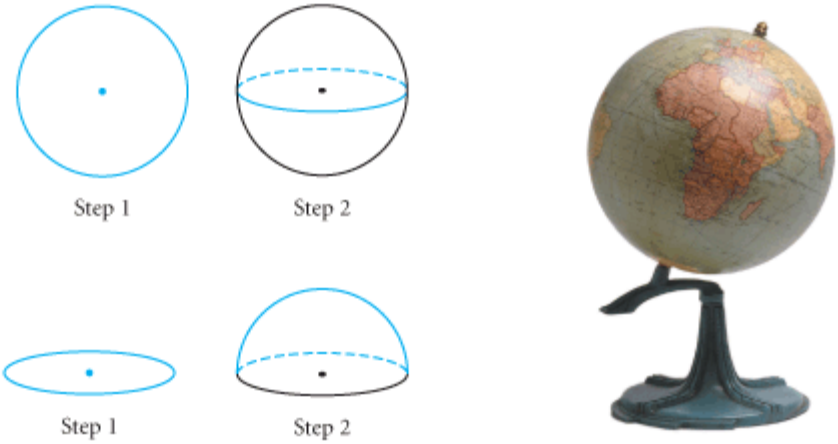
Prism



Pyramid



Sphere



Hemisphere

Solid geometry also involves visualizing points and lines in space. In the following investigation, you will have to visualize relationships between geometric figures in a plane and in space.

Investigation
Space Geometry

Step 1 Make a sketch or use physical objects to demonstrate each statement in the list below.

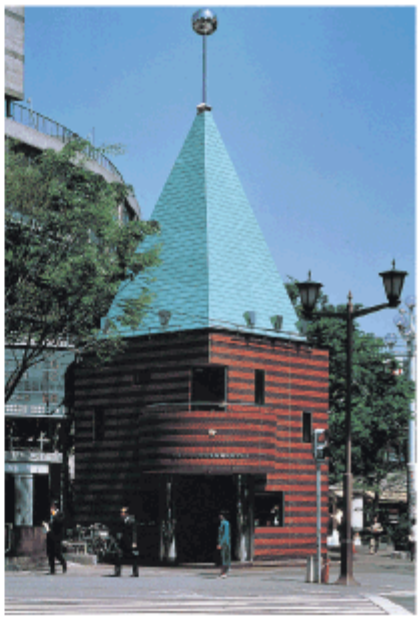
Step 2 Work with your group to determine whether each statement is true or false. If the statement is false, draw a picture and explain why it is false.

1. For any two points, there is exactly one line that can be drawn through them.
2. For any line and a point not on the line, there is exactly one plane that can contain them.
3. For any two lines, there is exactly one plane that contains them.
4. If two coplanar lines are both perpendicular to a third line in the same plane, then the two lines are parallel.
5. If two planes do not intersect, then they are parallel.
6. If two lines do not intersect, then they are parallel.
7. If a line is perpendicular to two lines in a plane, and the line is not contained in the plane, then the line is perpendicular to the plane.

EXERCISES

For Exercises 1–6, draw each figure. Study the drawing tips provided on the previous page before you start.

1. Cylinder
2. Cone
3. Prism with a hexagonal base
4. Sphere
5. Pyramid with a heptagonal base
6. Hemisphere
7. The photo at right shows a prism-shaped building with a pyramid roof and a cylindrical porch. Draw a cylindrical building with a cone roof and a prism-shaped porch.



A police station, or *koban*, in Tokyo, Japan

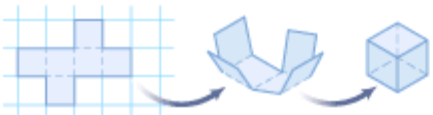
For Exercises 8 and 9, make a drawing to scale of each figure. Use isometric dot paper. Label each figure. (For example, in Exercise 8, draw the solid so that the dimensions measure 2 units by 3 units by 4 units, then label the figure with meters.)

8. A rectangular solid 2 m by 3 m by 4 m, sitting on its biggest face. (h)
9. A rectangular solid 3 inches by 4 inches by 5 inches, resting on its smallest face. Draw lines on the three visible surfaces showing how you can divide the solid into cubic-inch boxes. How many such boxes will fit in the solid? (h)

For Exercises 10–12, use isometric dot paper to draw the figure shown.

10. 
11. 
12. 

A **net** is a two-dimensional pattern that you can cut and fold to form a three-dimensional figure. Another visual thinking skill you will need is the ability to visualize nets being folded into solid objects and geometric solids being unfolded into nets. The net below left can be folded into a cube and the net below right can be folded into a pyramid.

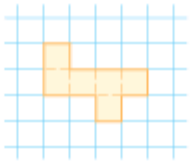


Net for a cube

Net for a square-based pyramid

13. Which net(s) will fold to make a cube?

A.

B. 

C.

D.

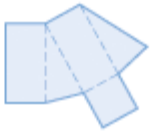
E.

F.

For Exercises 14–17, match the net with its geometric solid.

14.

15. 

16. 

17. 

A. 

B. 

C.

D. 

When a solid is cut by a plane, the resulting two-dimensional figure is called a **section.** For Exercises 18 and 19, sketch the section formed when each solid is sliced by the plane, as shown.

18. 

19. 

Slicing a block of clay reveals a section of the solid. Here, the section is a rectangle.

All of the statements in Exercises 20–27 are true except for two. Make a sketch to demonstrate each true statement. For each false statement, draw a sketch and explain why it is false.

20. Only one plane can pass through three noncollinear points.

21. If a line intersects a plane that does not contain the line, then the intersection is exactly one point.

22. If two lines are perpendicular to the same line, then they are parallel. (h)

23. If two different planes intersect, then their intersection is a line.

Physical models can help you visualize the intersections of lines and planes in space. Can you see examples of intersecting lines in this photo? Parallel lines? Planes? Points?

24. If a line and a plane have no points in common, then they are parallel.

25. If a plane intersects two parallel planes, then the lines of intersection are parallel.

26. If three planes intersect, then they divide space into six parts.

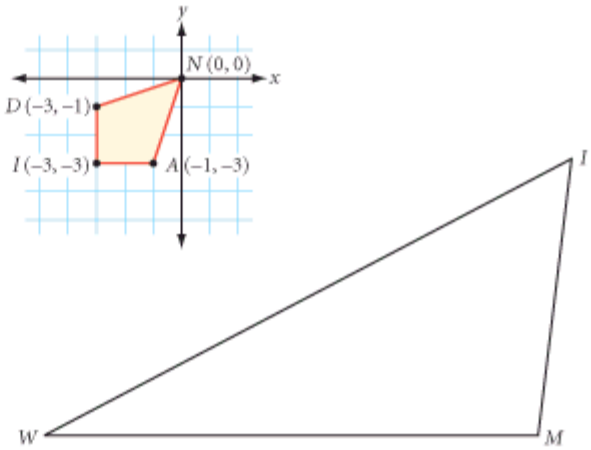
27. If two lines are perpendicular to the same plane, then they are parallel to each other.

Review

28. If the kite *DIAN* were rotated 90° clockwise about the origin, to what location would point *A* be relocated?

29. Use your ruler to measure the perimeter of △*WIM* (in centimeters) and your protractor to measure the largest angle.

30. Use your geometry tools to draw a triangle with two sides of length 8 cm and length 13 cm and the angle between them measuring 120°.



IMPROVING YOUR VISUAL THINKING SKILLS

Equal Distances

Here's a real challenge:

Show four points *A*, *B*, *C*, and *D* so that $AB = BC = AC = AD = BD = CD$.