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LESSON 1.7

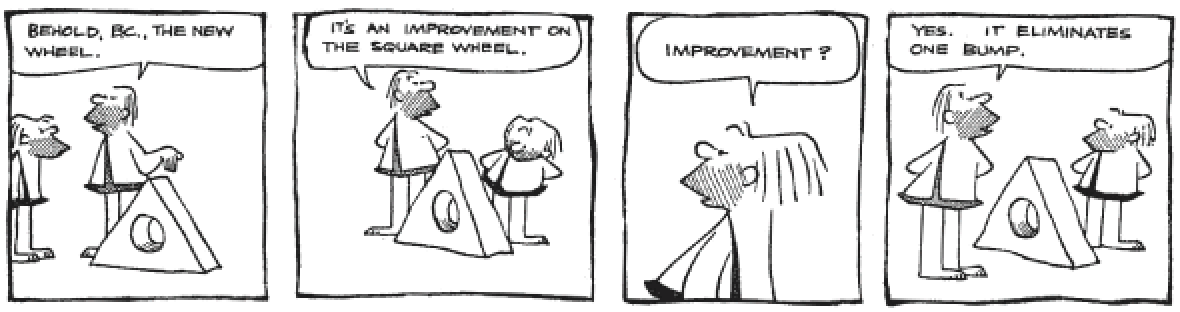
Circles

I can never remember things I didn't understand in the first place.
AMY TAN

Unless you walked to school this morning, you arrived on a vehicle with circular wheels.

A **circle** is the set of all points in a plane at a given distance from a given point. The given distance is called the **radius** and the given point is called the **center.** You name a circle by its center. The circle on the bicycle wheel, with center O, is called circle O. When you see a dot at the center of a circle, you can assume that it represents the center point.

A segment from the center to a point on the edge of the circle is called a radius. Its length is also called the radius. A bicycle wheel is a physical model of a circle, and one spoke is a close physical model of a radius.



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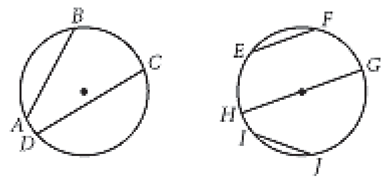
Science CONNECTION

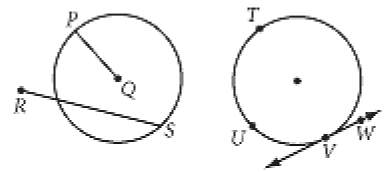
A pebble dropped in a pond sends out circular ripples. These waves radiate from the point where the pebble hits the water in all directions at the same speed, so every point is equally distant from the center. This unique property of circles appears in many other real-world contexts, such as radio waves sent from an antenna, seismic waves moving from the center of an earthquake, or sand draining out of a hole.

Investigation
Defining Circle Terms

Step 1 Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions as a class and add them to your definition list. In your notebook, draw and label a figure to illustrate each definition.

Chord

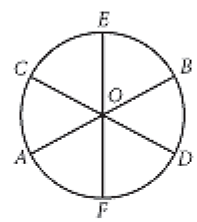


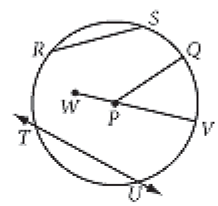


Chords:
$\overline{AB}$, $\overline{CD}$, $\overline{EF}$, $\overline{GH}$, and $\overline{IJ}$

Not chords:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overrightarrow{VW}$

Diameter

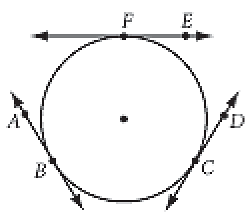


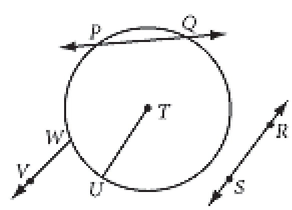


Diameters:
$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$

Not diameters:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overline{VW}$

Tangent





Tangents:
$\overline{AB}$, $\overline{CD}$, and $\overline{EF}$

Not tangents:
$\overline{PQ}$, $\overline{RS}$, $\overline{TU}$, and $\overline{VW}$

Note: You can say $\overleftrightarrow{AB}$ is a tangent, or you can say $\overleftrightarrow{AB}$ is tangent to circle O. The point where the tangent touches the circle is called the **point of tangency.**

Step 2 Can a chord of a circle also be a diameter of the circle? Can it be a tangent? Explain why or why not.

Step 3 Can two circles be tangent to the same line at the same point? Draw a sketch and explain.

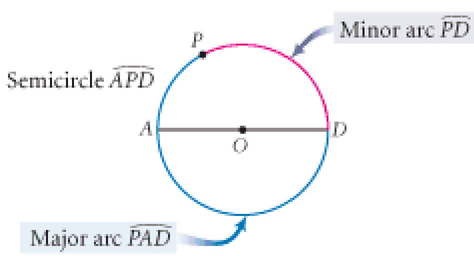
If two or more circles have the same radius, they are **congruent circles.** If two or more coplanar circles share the same center, they are **concentric circles.** All the CDs represent congruent circles, but if you look closely at each CD, you can also see concentric circles.

Congruent circles

Concentric circles

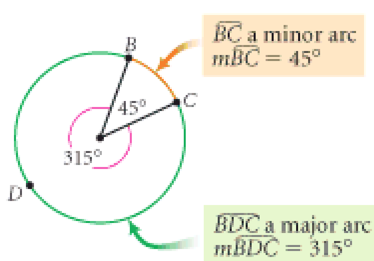
An **arc** of a circle is two points on the circle and a continuous (unbroken) part of the circle between the two points. The two points are called the **endpoints** of the arc.

You write arc AB as $\overset{\frown}{AB}$ or $\overset{\frown}{BA}$. You classify arcs into three types: semicircles, minor arcs, and major arcs. A **semicircle** is an arc of a circle whose endpoints are the endpoints of a diameter. A **minor arc** is an arc of a circle that is smaller than a semicircle. A **major arc** is an arc of a circle that is larger than a semicircle. You can name minor arcs with the letters of the two endpoints. For semicircles and major arcs, you need three points to make clear which arc you mean—the first and last letters are the endpoints and the middle letter is any other point on the arc.



Try to name another minor arc and another major arc in this diagram. Why are three letters needed to name a major arc?

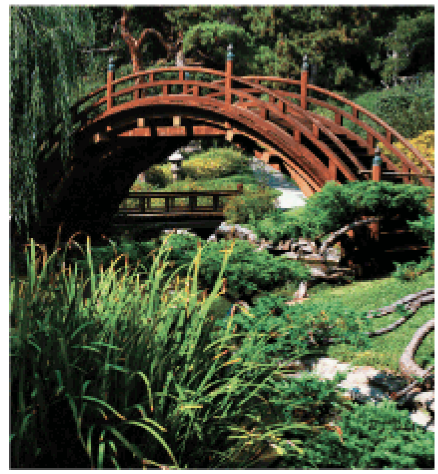
Arcs have a degree measure, just as angles do. A full circle has an arc measure of 360°, a semicircle has an arc measure of 180°, and so on. The **arc measure** of a minor arc is the same as the measure of the **central angle**, the angle with its vertex at the center of the circle, and sides passing through the endpoints of the arc. The measure of a major arc is the same as the reflex measure of the central angle.



EXERCISES

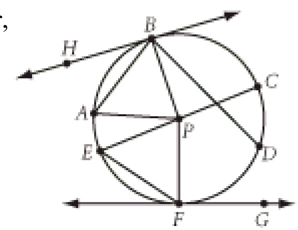
1. In the photos below, identify the physical models that represent a circle, a radius, a chord, a tangent, and an arc of a circle.

Circular irrigation on a farm



Japanese wood bridge

For Exercises 2–9, use the diagram at right. Points *E*, *P*, and *C* are collinear, and *P* is the center of the circle.



2. Name three chords.

3. Name one diameter.

4. Name five radii.

5. Name five minor arcs.

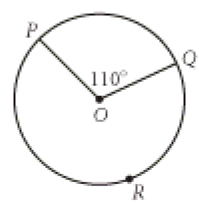
6. Name two semicircles.

7. Name two major arcs.

8. Name two tangents.

9. Name a point of tangency.

10. Name two types of vehicles that use wheels, two household appliances that use wheels, and two uses of the wheel in the world of entertainment.



11. In the figure at right, what is $m\overset{\frown}{PQ}$? $m\overset{\frown}{PRQ}$?

12. Use your compass and protractor to make an arc with measure 65°. Now make an arc with measure 215°. Label each arc with its measure.

13. Name two places or objects where concentric circles appear. Bring an example of a set of concentric circles to class tomorrow. You might look in a magazine for a photo or make a copy of a photo from a book (but not this book!).

14. Sketch two circles that appear to be concentric. Then use your compass to construct a pair of concentric circles.

15. Sketch circle *P*. Sketch a triangle inside circle *P* so that the three sides of the triangle are chords of the circle. This triangle is "inscribed" in the circle. Sketch another circle and label it *Q*. Sketch a triangle in the exterior of circle *Q* so that the three sides of the triangle are tangents of the circle. This triangle is "circumscribed" about the circle.

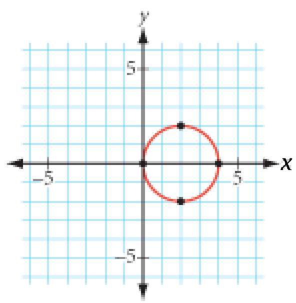
16. Use your compass to construct two circles with the same radius intersecting at two points. Label the centers P and Q. Label the points of intersection of the two circles A and B. Construct quadrilateral $PAQB$. What type of quadrilateral is it?

17. Do you remember the daisy construction from Chapter 0? Construct a circle with radius s. With the same compass setting, divide the circle into six congruent arcs. Construct the chords to form a regular hexagon inscribed in the circle. Construct radii to each of the six vertices. What type of triangle is formed? What is the ratio of the perimeter of the hexagon to the diameter of the circle?

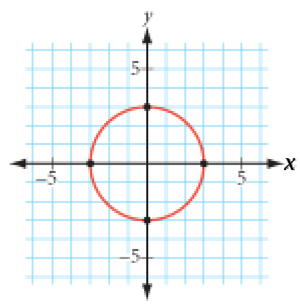
18. Sketch the path made by the midpoint of a radius of a circle if the radius is rotated about the center.

For Exercises 19–21, use the ordered pair rule shown to relocate the four points on the given circle. Can the four new points be connected to create a new circle? Does the new figure appear congruent to the original circle?

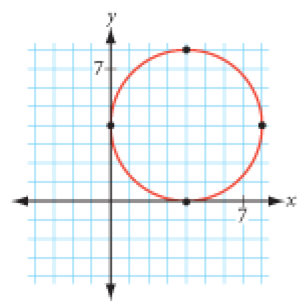
19. $(x, y) \rightarrow (x - 1, y + 2)$



20. $(x, y) \rightarrow (2x, 2y)$



21. $(x, y) \rightarrow (2x, y)$



Review

22. If point D is in the interior of $\angle CAB$, thenk $m\angle CAD + m\angle DAB = m\angle CAB$. called **angle addition.** Solve the following problem and explain how it is related to angle addition.

You have a slice of pizza with a central angle that measures 140° that you want to share with your friend. She cuts it through the vertex into two slices. You choose one slice that measures 60°. How many degrees are in the other slice?

For Exercises 23–26, draw each kind of triangle or write "not possible" and explain why. Use your geometry tools to make your drawings as accurate as possible.

23. Isosceles right triangle

24. Scalene isosceles triangle

25. Scalene obtuse triangle

26. Isosceles obtuse triangle

27. Earth takes 365.25 days to travel one full revolution around the Sun. By approximately how many degrees does the Earth travel each day in its orbit around the Sun?

28. Earth completes one full rotation each day, making the Sun appear to rise and set. If the Sun passes directly overhead, by how many degrees does its position in the sky change every hour?

For Exercises 29–37, sketch, label, and mark the figure or write "not possible" and explain why.

29. Obtuse scalene triangle *FAT* with $m\angle FAT = 100°$

30. Trapezoid *TRAP* with $\overline{TR} \parallel \overline{AP}$ and $\angle TRA$ a right angle

31. Two different (noncongruent) quadrilaterals with angles of 60°, 60°, 120°, and 120°

32. Equilateral right triangle

33. Right isosceles triangle *RGT* with $RT = GT$ and $m\angle RTG = 90°$

34. An equilateral triangle with perimeter $12a + 6b$

35. Two triangles that are not congruent, each with angles measuring 50° and 70°

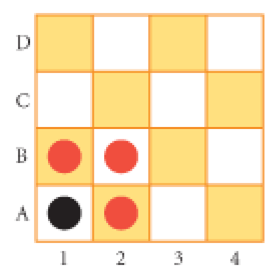
36. Rhombus *EQUI* with perimeter $8p$ and $m\angle IEQ = 55°$

37. Kite *KITE* with $TE = 2EK$ and $m\angle TEK = 120°$

IMPROVING YOUR REASONING SKILLS

Checkerboard Puzzle

1. Four checkers—three red and one black—are arranged on the corner of a checkerboard, as shown at right. Any checker can jump any other checker. The checker that was jumped over is then removed. With exactly three horizontal or vertical jumps, remove all three red checkers, leaving the single black checker. Record your solution.



2. Now, with exactly seven horizontal or vertical jumps, remove all seven red checkers, leaving the single black checker. Record your solution.