Student Web Links @ Keymath.com

LESSON 1.4

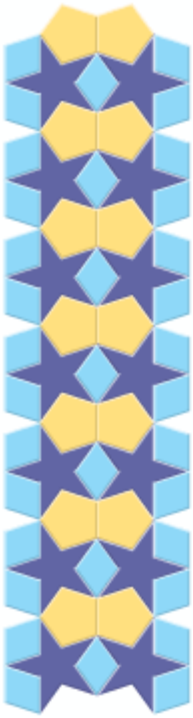
Polygons

There are two kinds of people in this world: those who divide everything into two groups, and those who don't.
KENNETH BOULDING

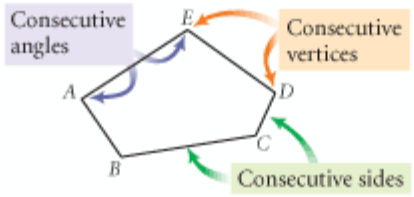
A **polygon** is a closed figure in a plane, formed by connecting line segments endpoint to endpoint with each segment intersecting exactly two others. Each line segment is called a **side** of the polygon. Each endpoint where the sides meet is called a **vertex** of the polygon.

Polygons

Not Polygons



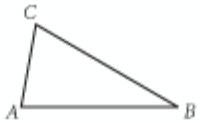
You classify a polygon by the number of sides it has. Familiar polygons have specific names, listed in this table. The ones without specific names are called *n*-sided polygons, or *n*-gons. For instance, you call a 25-sided polygon a 25-gon.



Sides	Name
3	**Triangle**
4	**Quadrilateral**
5	**Pentagon**
6	**Hexagon**
7	**Heptagon**
8	**Octagon**
9	**Nonagon**
10	**Decagon**
11	**Undecagon**
12	**Dodecagon**
n	*n*-gon

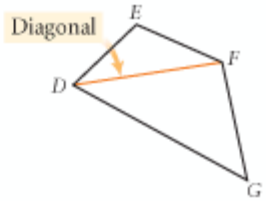
To name a polygon, list the vertices in consecutive order. You can name the pentagon above pentagon *ABCDE*. You can also call it *DCBAE*, but not *BCAED*. When the polygon is a triangle, you use the triangle symbol. For example, $\triangle ABC$ means triangle *ABC*.

Pentagon *ABCDE*

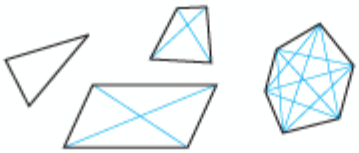


$\triangle ABC$

A **diagonal** of a polygon is a line segment that connects two nonconsecutive vertices.



A polygon is **convex** if no diagonal is outside the polygon. A polygon is **concave** if at least one diagonal is outside the polygon.



Convex polygons: All diagonals are inside

Concave polygons: One or more diagonals are outside

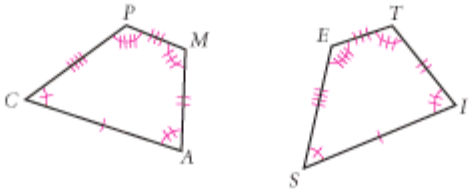
Recall that two segments or two angles are congruent if and only if they have the same measures. Two polygons are **congruent** if and only if they are exactly the same size and shape. "If and only if" means that the statements work both ways.

How does the shape of the framework of this Marc Chagall (1887–1985) stained glass window support the various shapes of the design?

If polygons are congruent, then corresponding sides and angles are congruent.

If corresponding sides and angles are congruent, then polygons are congruent.

For example, if quadrilateral *CAMP* is congruent to quadrilateral *SITE*, then their four pairs of corresponding angles and four pairs of corresponding sides are also congruent. When you write a statement of congruence, always write the letters of the corresponding vertices in an order that shows the correspondences.

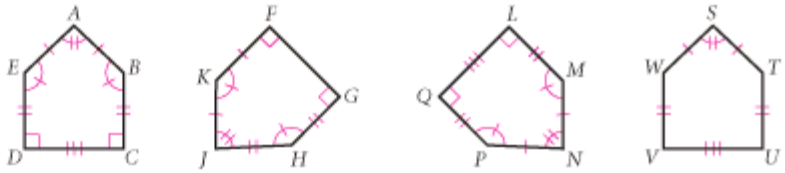


$CAMP \cong SITE$

EXAMPLE

Which polygon is congruent to *ABCDE*?

$ABCDE \cong \underline{?}$



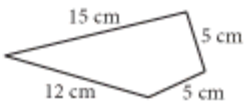
Solution

Polygons *JKFGH* and *ABCDE* have all corresponding angles congruent, but not all corresponding sides. Polygons *STUVW* and *ABCDE* have all corresponding sides congruent, but not all corresponding angles.

All corresponding sides and angles must be congruent, so $ABCDE \cong NPQLM$.

You could also say $ABCDE \cong NMLQP$ because all the congruent parts would still match.

The **perimeter** of a polygon equals the sum of the lengths of its sides. Perimeter measures the length of the boundary of a two-dimensional figure.

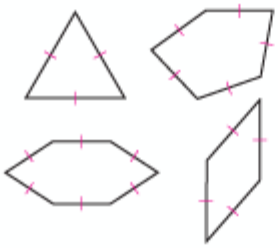


The quadrilateral at right has perimeter 37 cm.

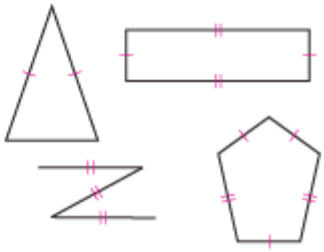
Investigation
Special Polygons

Write a good definition of each boldfaced term. Discuss your definitions with others in your group. Agree on a common set of definitions for your class and add them to your definitions list. In your notebook, draw and label a figure to illustrate each definition.

Equilateral Polygon

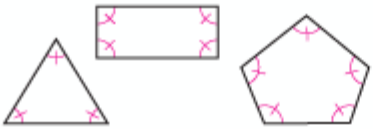


Equilateral polygons

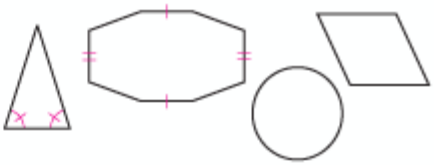


Not equilateral polygons

Equiangular Polygon

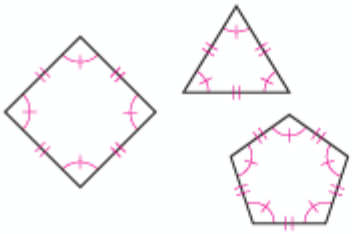


Equiangular polygons

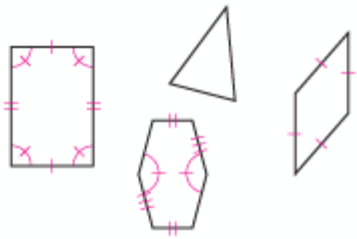


Not equiangular polygons

Regular Polygon



Regular polygons



Not regular polygons

EXERCISES

For Exercises 1–3, draw an example of each polygon.

1. Quadrilateral **2.** Dodecagon **3.** Octagon

For Exercises 4–7, classify each polygon. Assume that all sides are straight.

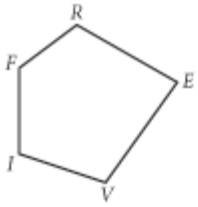
4. 

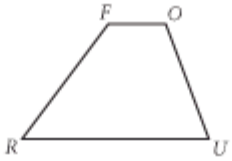
5. 

6. 

7. 

For Exercises 8–10, give one possible name for each polygon.

8. 

9. 

10. 

11. Write these definitions using the classify and differentiate method to fill in the blanks:

a. An octagon is _____ that _____.

b. A concave polygon is _____ that _____.

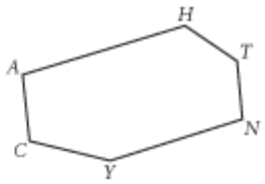
c. A 20-gon, also called an icosagon, is _____ that _____.

d. An equilateral polygon is _____ that _____.

12. Name a pair of consecutive angles and a pair of consecutive sides in the figure at right.

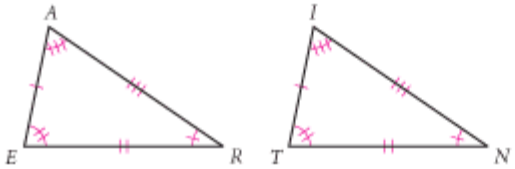
13. Draw a concave hexagon. How many diagonals does it have?

14. Name the diagonals of pentagon *ABCDE*.

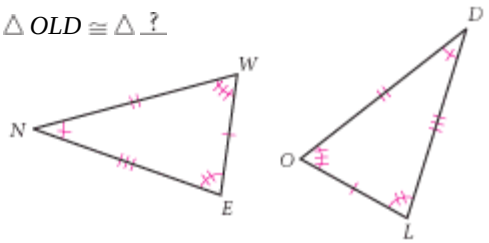


For Exercises 15 and 16, use the information given to name the triangle that is congruent to the first one.

15. $\triangle EAR \cong \triangle\ ?$ (h)



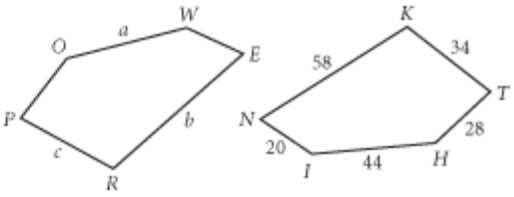
16. $\triangle OLD \cong \triangle\ ?$



17. In the figure at right, *THINK* $\cong$ *POWER*.

a. Find the measures *a*, *b*, and *c*.

b. If $m\angle P = 87°$ and $m\angle W = 165°$, which angles in *THINK* do you know? Write their measures.



18. If pentagon *FIVER* is congruent to pentagon *PANCH*, then which side in pentagon *FIVER* is congruent to side $\overline{PA}$? Which angle in pentagon *PANCH* is congruent to $\angle IVE$?

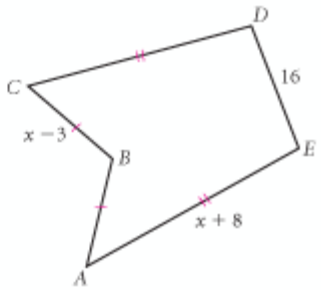
19. Use your geometry tools to draw a convex hexagon with two consecutive sides measuring 5 cm and three consecutive angles measuring 130°.

20. Draw an equilateral concave pentagon. Then draw an equiangular convex pentagon.

21. Each side of a regular dodecagon measures 7 in. Find the perimeter.

22. The perimeter of an equilateral octagon is 42 cm. Find the length of each side.

23. The perimeter of *ABCDE* is 94 m. Find the lengths of segments *AB* and *CD*.

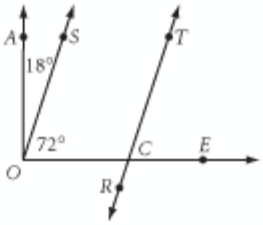


Review

24. Name a pair of complementary angles and a pair of vertical angles in the figure at right.

25. Draw $\overleftrightarrow{AB}$, $\overleftrightarrow{CD}$, and $\overleftrightarrow{EF}$ with $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$ and $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$.

26. Draw a counterexample to show that this statement is false: "If a rectangle has perimeter 50 meters, then a pair of consecutive sides measures 10 meters and 15 meters."



27. Is it possible for four lines in a plane to have exactly zero points of intersection? One point? Two points? Three points? Four points? Five points? Six points? Draw a figure to support each of your answers.

IMPROVING YOUR VISUAL THINKING SKILLS

Coin Swap II

Arrange three dimes and three pennies on a grid of seven squares, as shown. Follow the same rules as in Coin Swap I on page 46 to switch the position of the three dimes and three pennies in exactly 15 moves. Record your solution by listing in order which coin is moved. For example, your list might begin PDP. . . .