LESSON 1.3

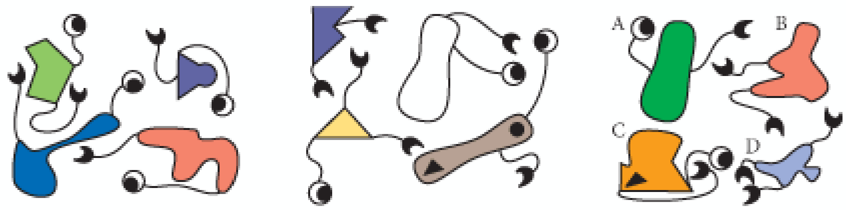
What's a Widget?

Good definitions are very important in geometry. In this lesson you will write your own geometry definitions.

Which creatures in the last group are Widgets?

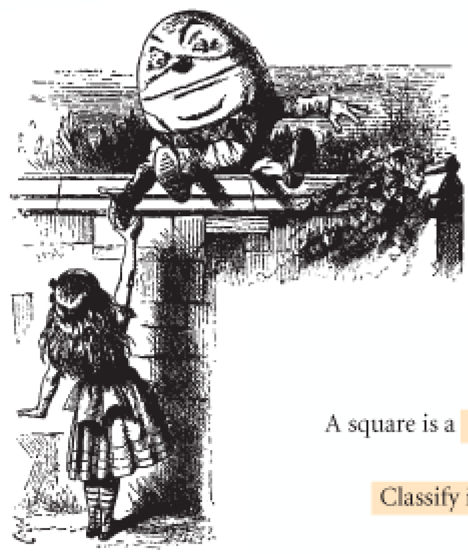
"When I use a word," Humpty replied in a scornful tone, "it means just what I choose it to mean—neither more nor less." "The question is," said Alice, "whether you can make a word mean so many different things."

LEWIS CARROLL



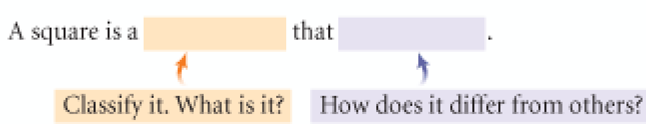
Widgets — Not Widgets — Which are Widgets?

You might have asked yourself, "What things do all the Widgets have in common, and what things do Widgets have that others do not have?" In other words, what characteristics make a Widget a Widget? They all have colorful bodies with nothing else inside; two tails—one like a crescent moon, the other like an eyeball.



By observing what a Widget is and what a Widget isn't, you identified the characteristics that distinguish a Widget from a non-Widget. Based on these characteristics, you should have selected A as the only Widget in the last group. This same process can help you write good definitions of geometric figures.

This statement defines a protractor: "A protractor is a geometry tool used to measure angles." First, you classify what it is (a geometry tool), then you say how it differs from other geometry tools (it is the one you use to measure angles). What should go in the blanks to define a square?



Once you've written a definition, you should test it. To do this, you look for a **counter example.** That is, try to create a figure that fits your definition but *isn't* what you're trying to define. If you can come up with a counterexample for your definition, you don't have a good definition.

EXAMPLE A

Everyone knows, "A square is a figure with four equal sides." What's wrong with this definition?

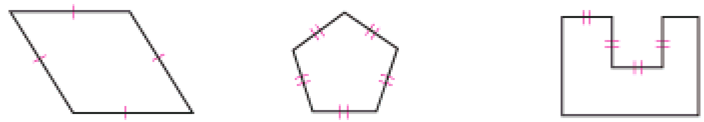
a. Sketch a counterexample. (You can probably find more than one!)

b. Write a better definition for a square.

▶ Solution

You probably noticed that “figure” is not specific enough to classify a square, and that “four equal sides” does not specify how it differs from the first counterexample shown below.

a. Three counterexamples are shown here, and you may have found others too.



b. One better definition is “A square is a 4-sided figure that has all sides congruent and all angles measuring 90 degrees.”

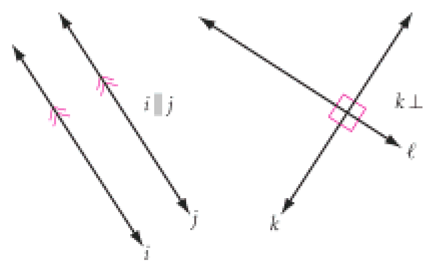
A restaurant counter example

Beginning Steps to Creating a Good Definition

1. **Classify** your term. What is it? (“A square is a 4-sided figure . . .”)
2. **Differentiate** your term. How does it differ from others in that class? (“. . . that has four congruent sides and four right angles.”)
3. **Test** your definition by looking for a counterexample.

Ready to write a couple of definitions? First, here are two more types of markings that are very important in geometry.

The same number of arrow marks indicates that lines are parallel. The symbol ∥ means “is parallel to.” A small square in the corner of an angle indicates that it measures 90°. The symbol ⊥ means “is perpendicular to.”



EXAMPLE B

Define these terms:

a. Parallel lines

b. Perpendicular lines

▶ Solution

Following these steps, classify and differentiate each term.

Classify. Differentiate.

a. Parallel lines are lines in the same plane that never meet.

b. Perpendicular lines are lines that meet at 90° angles.

Why do you need to say “in the same plane” for parallel lines, but not for perpendicular lines? Sketch or demonstrate a counterexample to show the following definition is incomplete: “Parallel lines are lines that never meet.” (Two lines that do not intersect and are noncoplanar are **skew lines**.)

Investigation

Defining Angles

Here are some examples and non-examples of special types of angles.

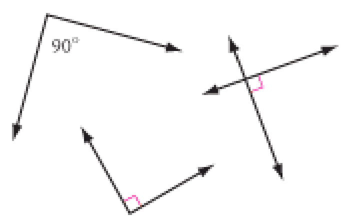
Step 1 Write a definition for each boldfaced term. Make sure your definitions highlight important differences.

Step 2 Trade definitions and test each other's definitions by looking for counterexamples.

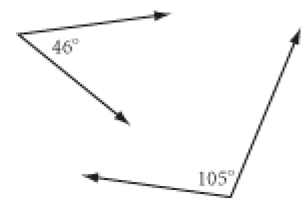
Step 3 If another group member finds a counterexample to one of your definitions, write a better definition. As a group, decide on the best definition for each term.

Step 4 As a class, agree on common definitions. Add these to your notebook. Draw and label a picture to illustrate each definition.

Right Angle



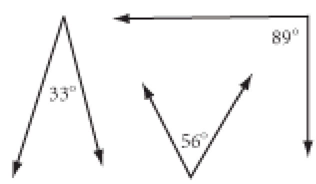
Right angles



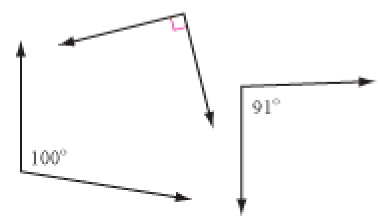
Not right angles

Notice the many congruent angles in this Navajo transitional Wedgeweave blanket. Are they right, acute, or obtuse angles?

Acute Angle

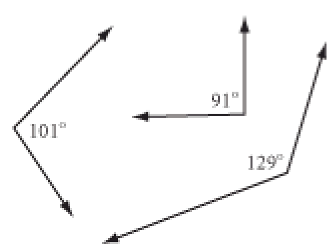


Acute angles

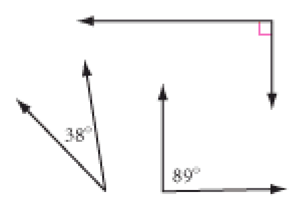


Not acute angles

Obtuse Angle



Obtuse angles



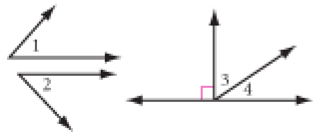
Not obtuse angles

keymath.com/DG

[You can also view the **Dynamic Geometry Exploration** Three Types of Angles at www.keymath.com/DG .]

Complementary Angles

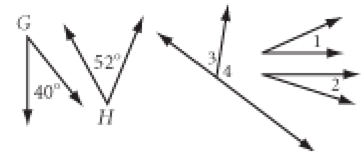
$m\angle 1 + m\angle 2 = 90°$



Pairs of complementary angles:

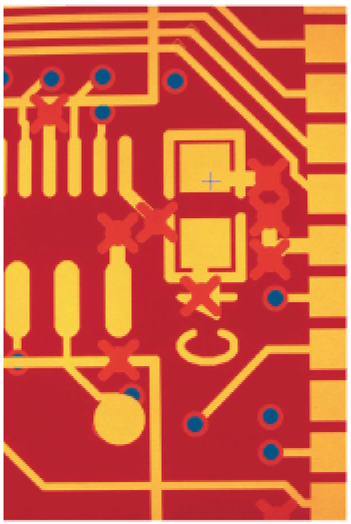
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

$m\angle 1 + m\angle 2 \neq 90°$



Not pairs of complementary angles:

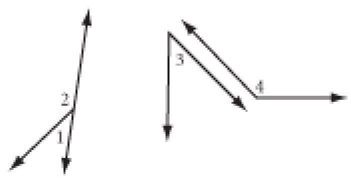
$\angle G$ and $\angle H$ $\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$



What types of angles or angle pairs do you see in this magnified view of a computer chip?

Supplementary Angles

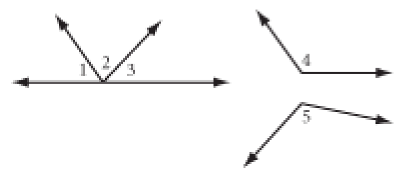
$m\angle 3 + m\angle 4 = 180°$



Pairs of supplementary angles:

$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$

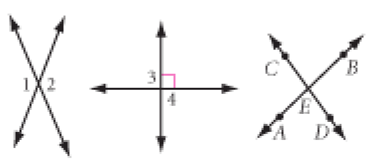
$m\angle 4 + m\angle 5 > 180°$



Not pairs of supplementary angles:

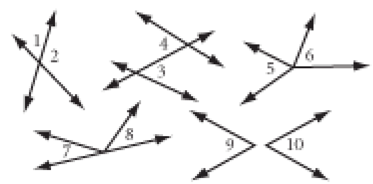
$\angle 1$, $\angle 2$, and $\angle 3$
$\angle 4$ and $\angle 5$

Vertical Angles



Pairs of vertical angles:

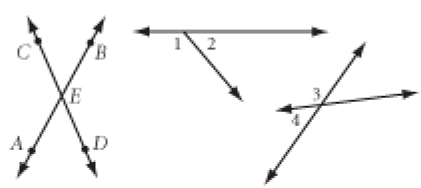
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle BEC$
$\angle AEC$ and $\angle DEB$



Not pairs of vertical angles:

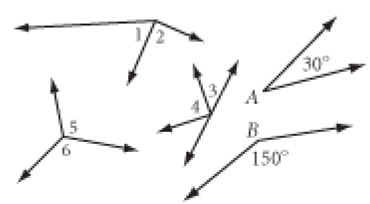
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle 7$ and $\angle 8$
$\angle 9$ and $\angle 10$

Linear Pair of Angles



Linear pairs of angles:

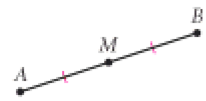
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle AED$ and $\angle AEC$
$\angle BED$ and $\angle DEA$



Not linear pairs of angles:

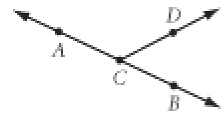
$\angle 1$ and $\angle 2$
$\angle 3$ and $\angle 4$
$\angle 5$ and $\angle 6$
$\angle A$ and $\angle B$

Often geometric definitions are easier to write if you refer to labeled figures. For example, you can define the midpoint of a line segment by saying: "Point *M* is the midpoint of segment *AB* if *M* is a point on segment *AB*, and *AM* is equal to *MB*."



EXAMPLE C Use a labeled figure to define a linear pair of angles.

Solution $\angle ACD$ and $\angle BCD$ form a linear pair of angles if point *C* is on $\overleftrightarrow{AB}$ and lies between points *A* and *B*.



Compare this definition with the one you wrote in the investigation. Can there be more than one correct definition?

The design of this African Kente cloth contains examples of parallel and perpendicular lines, obtuse and acute angles, and complementary and supplementary angle pairs. To learn about the significance of Kente cloth designs, visit www.keymath.com/DG.

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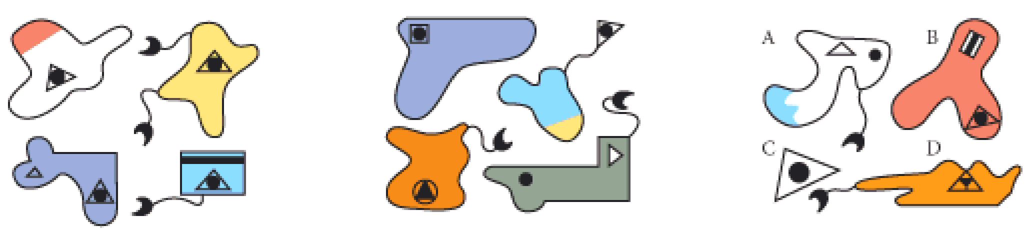
EXERCISES

For Exercises 1–8, draw and carefully label the figures. Use the appropriate marks to indicate right angles, parallel lines, congruent segments, and congruent angles. Use a protractor and a ruler when you need to.

1. Acute angle *DOG* with a measure of 45°
2. Right angle *RTE*
3. Obtuse angle *BIG* with angle bisector $\overrightarrow{IE}$
4. $\overleftrightarrow{DG} \parallel \overleftrightarrow{MS}$
5. $\overleftrightarrow{PE} \perp \overleftrightarrow{AR}$
6. Vertical angles *ABC* and *DBE*
7. Complementary angles $\angle A$ and $\angle B$ with $m\angle A = 40°$
8. Supplementary angles $\angle C$ and $\angle D$ with $m\angle D = 40°$
9. Which creatures in the last group below are Zoids? What makes a Zoid a Zoid?



Zoids Not Zoids Which are Zoids?

10. What are the characteristics of a good definition?
11. What is the difference between complementary and supplementary angles?

12. If $\angle X$ and $\angle Y$ are supplementary angles, are they necessarily a linear pair? Why or why not?

13. Write these definitions using the classify and differentiate method to fill in the blanks:

a. An acute angle is __________ that ___________.

b. Complementary angles are __________ that ___________.

c. A midpoint is __________ that ___________.

d. A protractor is __________ that ___________.

14. There is something wrong with this definition for a pair of vertical angles: "If $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at point P, then $\angle APC$ and $\angle BPD$ are a pair of vertical angles." Sketch a counterexample to show why it is not correct. Can you add a phrase to correct it?

For Exercises 15–24, four of the statements are true. Make a sketch or demonstrate each true statement. For each false statement, draw a counterexample.

15. For every line segment there is exactly one midpoint.

16. For every angle there is exactly one angle bisector.

17. If two different lines intersect, then they intersect at one and only one point.

18. If two different circles intersect, then they intersect at one and only one point.

19. Through a given point on a line, there is one and only one line perpendicular to the given line. (h)

20. In every triangle there is exactly one right angle.

21. Through a point not on a line, one and only one line can be constructed parallel to the given line.

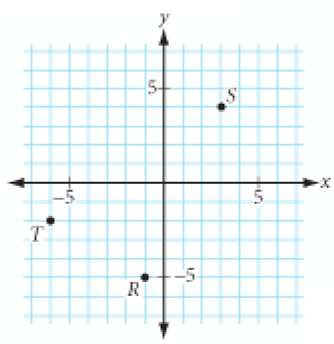
22. If $CA = AT$, then A is the midpoint of $\overline{CT}$.

23. If $m\angle D = 40°$ and $m\angle C = 140°$, then angles C and D are a linear pair.

24. If point A is not the midpoint of $\overline{CT}$, then $CA \neq AT$.

Review

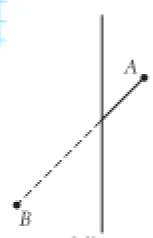
For Exercises 25 and 26, refer to the graph at right.

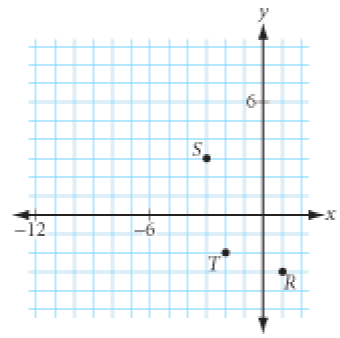


25. Find possible coordinates of a point P so that points P, T, and S are collinear.

26. Find possible coordinates of a point Q so that $\overleftrightarrow{QR} \parallel \overleftrightarrow{TS}$.

27. A *partial mirror* reflects some light and lets the rest of the light pass through. In the figure at right, half the light from point A passes through the partial mirror to point B. Copy the figure, then draw the outgoing angle for the light reflected from the mirror. What do you notice about the ray of reflected light and the ray of light that passes through? (h)





28. Find possible coordinates of points *A, B,* and *C* on the graph at right so that $\angle BAC$ is a right angle, $\angle BAT$ is an acute angle, $\angle ABS$ is an obtuse angle, and the points *C, T,* and *R* are collinear.

29. If *D* is the midpoint of $\overline{AC}$ and *C* is the midpoint of $\overline{AB}$, and $AD = 3\text{cm}$, what is the length of $\overline{AB}$?

30. If $\overrightarrow{BD}$ is the angle bisector of $\angle ABC$, $\overrightarrow{BE}$ is the angle bisector of $\angle ABD$, and $m\angle DBC = 24°$, what is $m\angle EBC$?

31. Draw and label a figure that has two congruent segments and three congruent angles. Mark the congruent angles and congruent segments.

32. Show how three lines in a plane can have zero, exactly one, exactly two, or exactly three points of intersection.

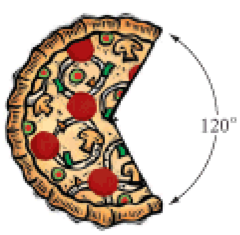
33. Show how it is possible for two triangles to intersect in one point, two points, three points, four points, five points, or six points, but not seven points. Show how they can intersect in infinitely many points.

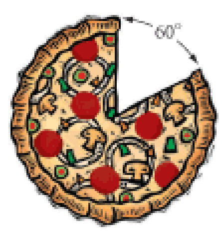
34. Each pizza is cut into slices from the center.

a. What fraction of the pizza is left?

b. What fraction of the pizza is missing?

c. If the pizza is cut into nine equal slices, how many degrees is each angle at the center of the pizza?





IMPROVING YOUR VISUAL THINKING SKILLS

Polyominoes

In 1953, United States mathematician Solomon Golomb introduced polyominoes at the Harvard Mathematics Club, and they have been played with and enjoyed throughout the world ever since. Polyominoes are shapes made by connecting congruent squares. The squares are joined together side to side. (A complete side must touch a complete side.) Some of the smaller polyominoes are shown below. There is only one monomino and only one domino, but there are two trominoes, as shown. There are five tetrominoes—one is shown. Sketch the other four.