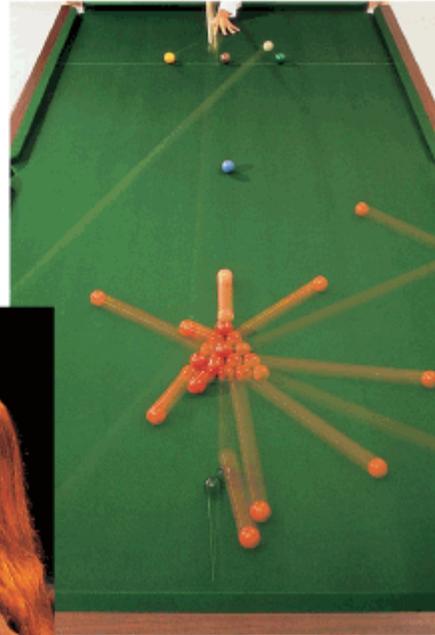


Inspiration is needed in geometry, just as much as in poetry.

ALEKSANDR PUSHKIN

Poolroom Math

People use angles every day. Plumbers measure the angle between connecting pipes to make a good fitting. Woodworkers adjust their saw blades to cut wood at just the correct angle. Air traffic controllers use angles to direct planes. And good pool players must know their angles to plan their shots.



Is the angle between the two hands of the wristwatch smaller than the angle between the hands of the large clock?

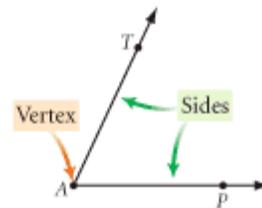
"Little Benji," the wristwatch



Big Ben at the Houses of Parliament in London, England

You can use the terms that you defined in Lesson 1.1 to write a precise definition of angle. An **angle** is formed by two rays that share a common endpoint, provided that the two rays are noncollinear. In other words, the rays cannot lie on the same line. The common endpoint of the two rays is the **vertex** of the angle. The two rays are the **sides** of the angle.

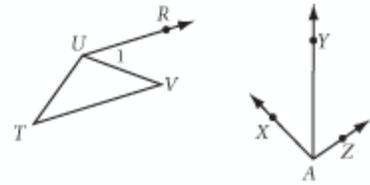
You can name the angle in the figure below angle TAP or angle PAT , or use the angle symbol and write $\angle TAP$ or $\angle PAT$. Notice that the vertex must be the middle letter, and the first and last letters each name a point on a different ray. Since there are no other angles with vertex A , you can also simply call this $\angle A$



EXAMPLE A | Name all the angles in these drawings.

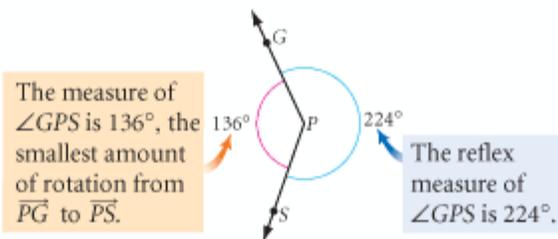
► **Solution**

The angles are $\angle T$, $\angle V$, $\angle TUV$, $\angle 1$, $\angle TUR$, $\angle XAY$, $\angle YAZ$, and $\angle XAZ$. (Did you get them all?) Notice that $\angle 1$ is a shorter way to name $\angle RUV$.

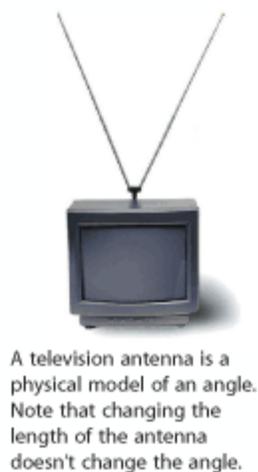


Which angles in Example A seem big to you? Which seem small?

The **measure of an angle** is the smallest amount of rotation about the vertex from one ray to the other, measured in **degrees**. According to this definition, the measure of an angle can be any value between 0° and 180° . The largest amount of rotation less than 360° between the two rays is called the **reflex measure of an angle**.



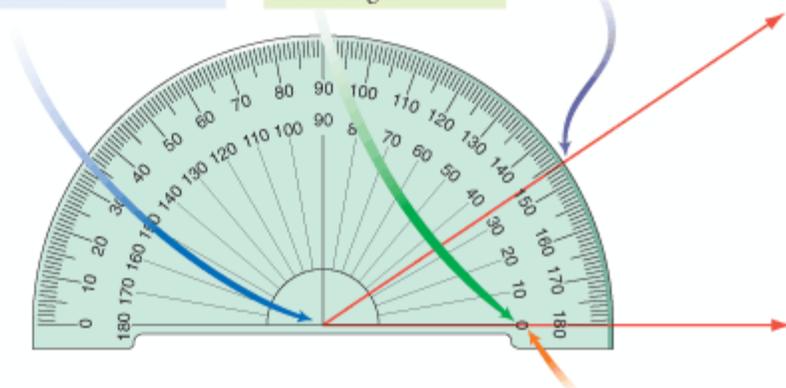
The geometry tool you use to measure an angle is a **protractor**. Here's how you use it.



Step 1: Place the center mark of the protractor on the vertex.

Step 2: Line up the 0-mark with one side of the angle.

Step 3: Read the measure on the protractor scale.



Step 4: Be sure you read the scale that has the 0-mark you are using! The angle in the diagram measures 34° and not 146° .

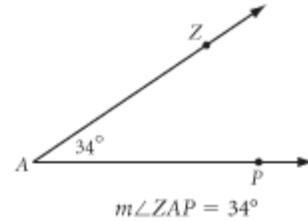
► For a visual tutorial on how to use a protractor, see the **Dynamic Geometry Exploration** at Protractor www.keymath.com/DG.

Career CONNECTION

In sports medicine, specialists may examine the healing rate of an injured joint by its angle of recovery. For example, a physician may assess how much physical therapy a patient needs by measuring the degree to which a patient can bend his or her ankle from the floor.

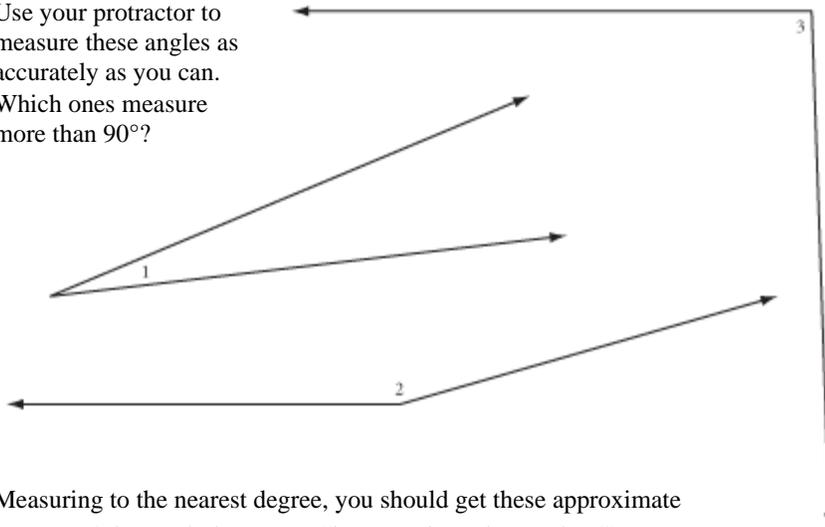


To show the measure of an angle, use an m before the angle symbol. For example, $m\angle ZAP = 34^\circ$ means the measure of $\angle ZAP$ is 34 degrees.



EXAMPLE B

Use your protractor to measure these angles as accurately as you can. Which ones measure more than 90° ?

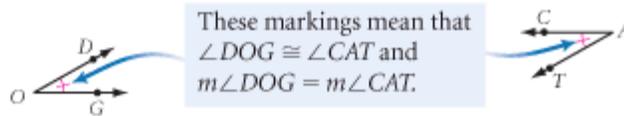


Solution

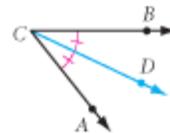
Measuring to the nearest degree, you should get these approximate answers. (The symbol \approx means “is approximately equal to.”)

$$\begin{array}{ll} m\angle 1 \approx 16^\circ & m\angle 3 \approx 92^\circ \\ m\angle 2 \approx 164^\circ & \angle 2 \text{ and } \angle 3 \text{ measure more than } 90^\circ. \end{array}$$

Two angles are **congruent** if and only if they have equal measure. You use identical markings to show that two angles in a figure are congruent.



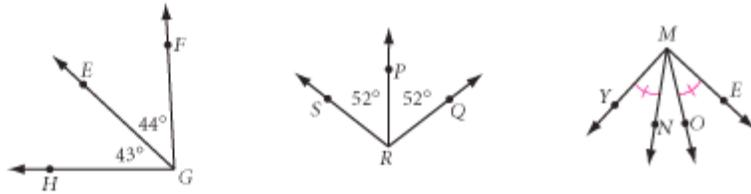
A ray is the **angle bisector** if it contains the vertex and divides the angle into two congruent angles. In the figure at right, \overline{CD} bisects $\angle ACB$ so that $\angle ACD \cong \angle BCD$.



EXAMPLE C

Look for angle bisectors and congruent angles in the figures below.

- Name each angle bisector and the angle it bisects.
- Name all the congruent angles in the figure. Use the congruence symbol and name the angles so there is no confusion about which angle you mean.

**► Solution**

- Use the angle bisector definition. $\angle SRP \cong \angle PRQ$, so RP bisects $\angle SRQ$.
- $\angle SRP \cong \angle PRQ$, $\angle YMN \cong \angle OME$, and $\angle YMO \cong \angle EMN$.



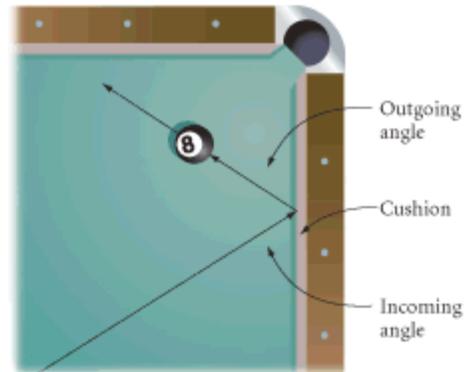
Investigation Virtual Pool

You will need

- the worksheet Poolroom Math
- a protractor

Pocket billiards, or pool, is a game of angles. When a ball bounces off the pool table's cushion, its path forms two angles with the edge of the cushion. The **incoming angle** is formed by the cushion and the path of the ball approaching the cushion.

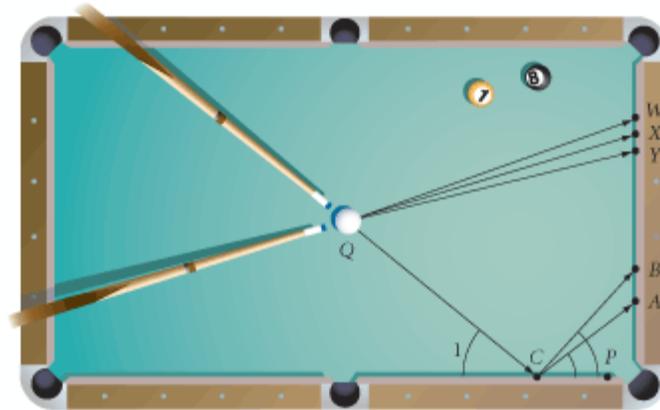
The **outgoing angle** is formed by the cushion and the path of the ball leaving the cushion. As it turns out, the measure of the outgoing angle equals the measure of the incoming angle.



Computer scientist Nesli O'Hare is also a professional pool player.



Use your protractor to study these shots.

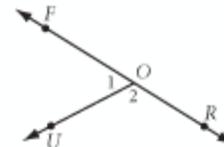
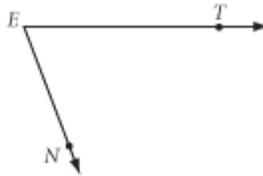


- Step 1 Use your protractor to find the measure of $\angle 1$. Which is the correct outgoing angle? Which point—A or B—will the ball hit?
- Step 2 Which point on the cushion—W, X, or Y—should the white ball hit so that the ray of the outgoing angle passes through the center of the 8-ball?
- Step 3 Compare your results with your group members' results. Does everyone agree?
- Step 4 How would you hit the white ball against the cushion so that the ball passes over the same spot on the way back?
- Step 5 How would you hit the ball so that it bounces off three different points on the cushions without ever touching cushion CP ?



EXERCISES

1. Name each angle in three different ways.



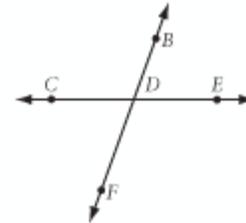
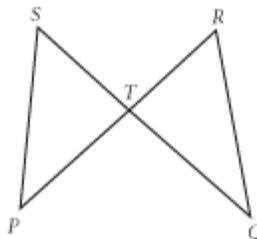
For Exercises 2–4, draw and label each angle.

2. $\angle TAN$

3. $\angle BIG$

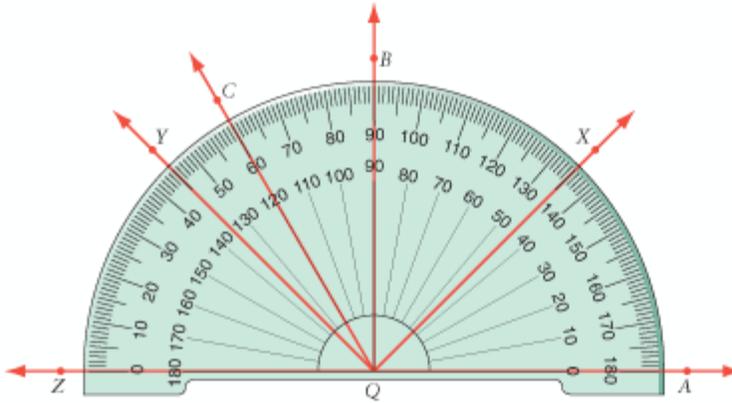
4. $\angle SML$

5. For each figure at right, list the angles that you can name using only the vertex letter.



6. Draw a figure that contains at least three angles and requires three letters to name each angle.

For Exercises 7–14, find the measure of each angle to the nearest degree.



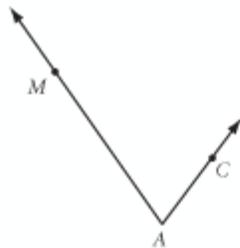
If an exercise has an **h** at the end, you can find a hint to help you in Hints for Selected Exercise at the back of the book.

7. $m\angle AQB \approx ?$ 8. $m\angle AQC \approx ?$ 9. $m\angle XQA \approx ?$ 10. $m\angle AQY \approx ?$
 11. $m\angle ZQY \approx ?$ 12. $m\angle ZQX \approx ?$ 13. $m\angle CQB \approx ?$ **h** 14. $m\angle XQY \approx ?$

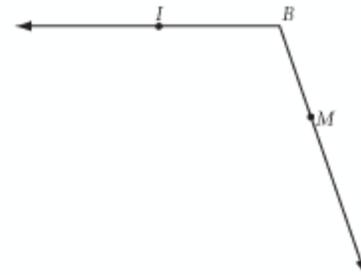
15. **Adjacent Angles** $\angle XQA$ and $\angle XQY$ share a vertex and a side. Taken together they form the larger angle $\angle AQY$. Compare their measures. Does $m\angle XQA + m\angle XQY = m\angle AQY$?

For Exercises 16–20, use your protractor to find the measure of the angle to the nearest degree.

16. $m\angle MAC \approx ?$



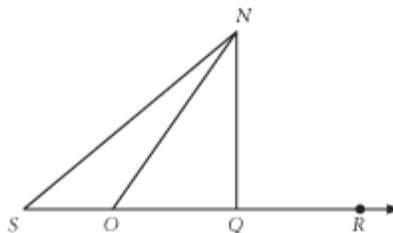
17. $m\angle IBM \approx ?$



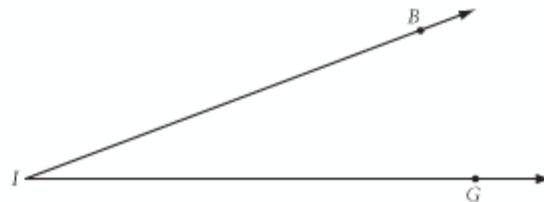
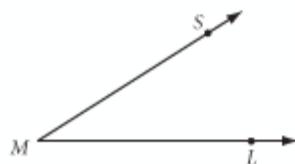
18. $m\angle S \approx ?$

19. $m\angle SON \approx ?$

20. $m\angle NOR \approx ?$



21. Which angle below has the greater measure, $\angle SML$ or $\angle BIG$? Why?



For Exercises 22–24, use your protractor to draw angles with these measures. Label them.

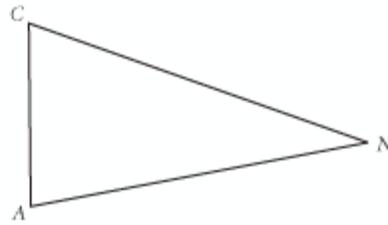
22. $m\angle A = 44^\circ$

23. $m\angle B = 90^\circ$

24. $m\angle CDE = 135^\circ$

25. Use your protractor to draw the angle bisector of $\angle A$ in Exercise 22 and the angle bisector of $\angle D$ in Exercise 24. Use markings to show that the two halves are congruent.

26. Copy triangle CAN shown at right. Use your protractor to find the angle bisector of $\angle A$. Label the point where it crosses CN point Y . Use your ruler to find the midpoint of CN and label it D . Are D and Y the same point?



For Exercises 27–29, draw a clock face with hands to show these times.

27. 3:30

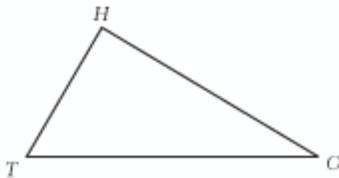
28. 3:40

29. 3:15

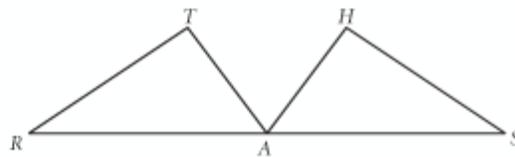
30. Give an example of a time when the angle made by the hands of the clock will be greater than 90° .

For Exercises 31–34, copy each figure and mark it with all the given information.

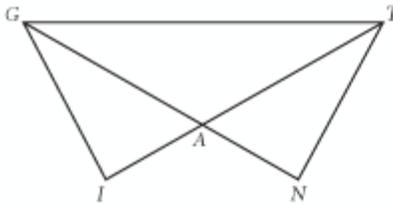
31. $TH = 6$
 $m\angle THO = 90^\circ$
 $OH = 8$



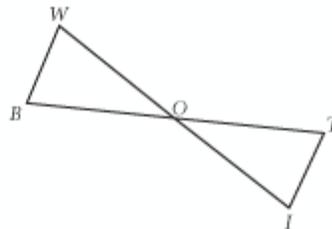
32. $RA = SA$
 $m\angle T = m\angle H$
 $RT = SH$



33. $AT = AG$ $\angle AGT \cong \angle ATG$
 $AI = AN$ $GI = TN$

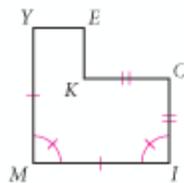


34. $\overline{BW} \cong \overline{TI}$ $\angle WBT \cong \angle ITB$
 $\overline{WO} \cong \overline{IO}$ $\angle BWO \cong \angle TIO$

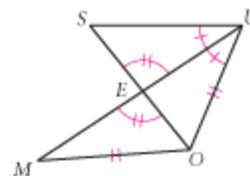


For Exercises 35 and 36, write down what you know from the markings. Do not use your protractor or your ruler.

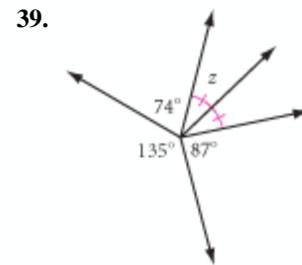
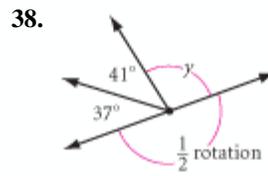
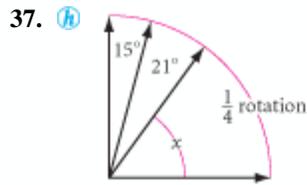
35. $MI = ?$
 $IC = ?$
 $m\angle M = ?$



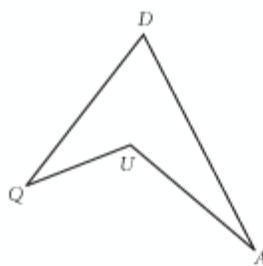
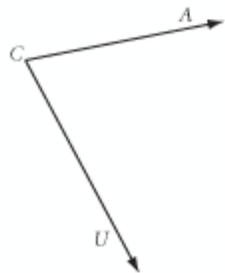
36. $\angle MEO \cong ?$
 $\angle SUE \cong ?$
 $OU = ?$



For Exercises 37–39, do not use a protractor. Recall from Chapter 0 that a complete rotation around a point is 360° . Find the angle measures represented by each letter.



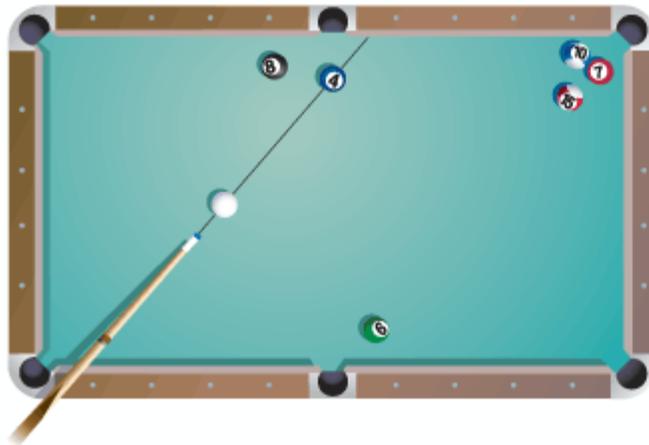
40. Use your protractor to determine the reflex measure of $\angle ACU$.



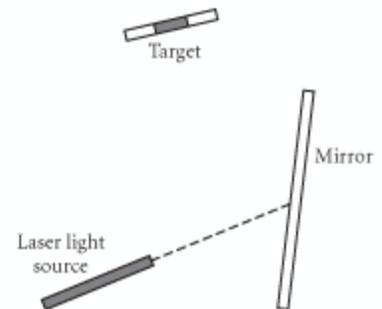
41. Use your protractor to determine the reflex measure of $\angle QUA$.

42. What is the relationship between the measure of an angle and the reflex measure of the angle?

43. If the 4-ball is hit as shown, will it go into the corner pocket? Find the path of the ball using only your protractor and straightedge.



44. The principle you just learned for billiard balls is also true for sound or radio waves bouncing off a surface or for a ray of light reflecting from a mirror. If you hold a laser light angled at the mirror as shown, will the light from the laser hit the target object? Explain.



Review

45. If points A , B , and C are collinear and B is between A and C , then $AB + BC = AC$. This is called **segment addition**. Solve the following problem and explain how it represents segment addition.

Podunkville, Smallville, and Gotham City lie along a straight highway with Smallville between the other two towns. If Podunkville and Smallville are 70 km apart and Smallville and Gotham City are 110 km apart, how far apart are Podunkville and Gotham City?

46. Use your ruler to draw a segment with length 12 cm. Then use your ruler to locate the midpoint. Label and mark the figure.
47. The balancing point of an object is called its *center of gravity*. Where is the center of gravity of a thin, rodlike piece of wire or tubing? Copy the thin wire shown below onto your paper. Mark the balance point or center of gravity.



48. Explain the difference between $MS = DG$ and $\overline{MS} \cong \overline{DG}$.
49. Use your ruler and protractor to draw a triangle with angle measures 40° and 70° . Explain your method. Can you draw a second triangle with these two angle measures that looks different from the first?



How is the center of gravity incorporated into the design of this structure?

IMPROVING YOUR VISUAL THINKING SKILLS

Coin Swap I

Arrange two dimes and two pennies on a grid of five squares, as shown. Your task is to switch the position of the two dimes and two pennies in exactly eight moves. A coin can slide into an empty square next to it, or it can jump over one coin into an empty space. Record your solution by drawing eight diagrams that show the moves.

