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LESSON 1.1

Building Blocks of Geometry

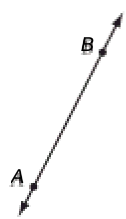
Nature's Great Book is written in mathematical symbols.
GALILEO GALILEI

Three building blocks of geometry are points, lines, and planes. A **point** is the most basic building block of geometry. It has no size. It has only location. You represent a point with a dot, and you name it with a capital letter. The point shown below is called *P*.

Mathematical model of a point

A tiny seed is a physical model of a point. A point, however, is smaller than any seed that ever existed.

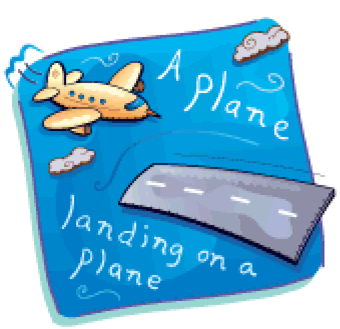
A **line** is a straight, continuous arrangement of infinitely many points. It has infinite length, but no thickness. It extends forever in two directions. You name a line by giving the letter names of any two points on the line and by placing the line symbol above the letters, for example, $\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$.

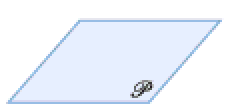


Mathematical model of a line

A piece of spaghetti is a physical model of a line. A line, however, is longer, straighter, and thinner than any piece of spaghetti ever made.

A **plane** has length and width, but no thickness. It is like a flat surface that extends infinitely along its length and width. You represent a plane with a four-sided figure, like a tilted piece of paper, drawn in perspective. Of course, this actually illustrates only part of a plane. You name a plane with a script capital letter, such as $\mathcal{P}$

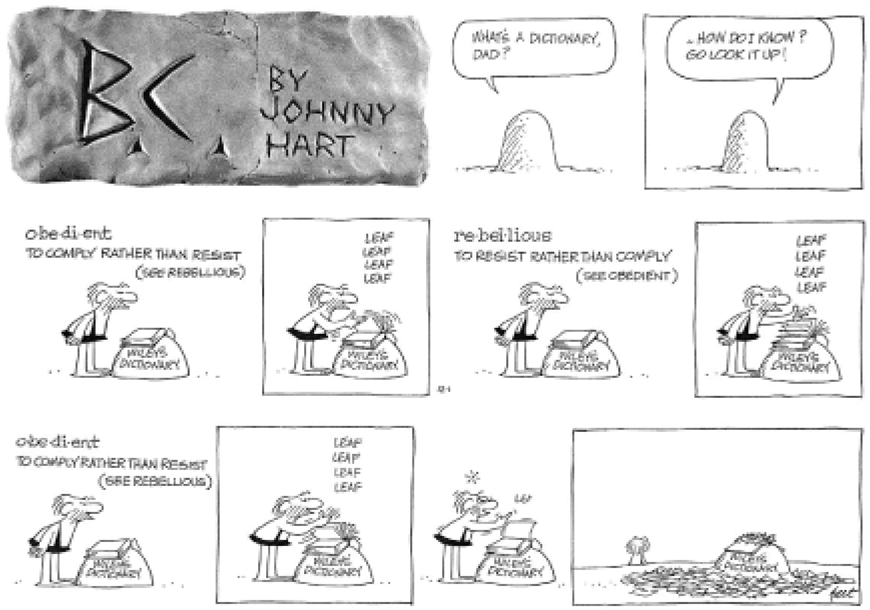




Mathematical model of a plane

A flat piece of rolled-out dough is a physical model of a plane. A plane, however, is broader, wider, and thinner than any piece of dough you could ever roll.

It can be difficult to explain what points, lines, and planes are even though you may recognize them. Early mathematicians tried to define these terms.



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The ancient Greeks said, “A point is that which has no part. A line is breadthless length.” The Mohist philosophers of ancient China said, “The line is divided into parts, and that part which has no remaining part is a point.” Those definitions don’t help much, do they?

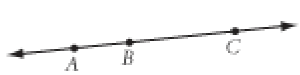
A **definition** is a statement that clarifies or explains the meaning of a word or a phrase. However, it is impossible to define point, line, and plane without using words or phrases that themselves need definition. So these terms remain undefined. Yet, they are the basis for all of geometry.

Using the undefined terms *point, line,* and *plane,* you can define all other geometry terms and geometric figures. Many are defined in this book, and others will be defined by you and your classmates.

Keep a definition list in your notebook, and each time you encounter new geometry vocabulary, add the term to your list. Illustrate each definition with a simple sketch.

Here are your first definitions. Begin your list and draw sketches for all definitions.

Collinear means on the same line.

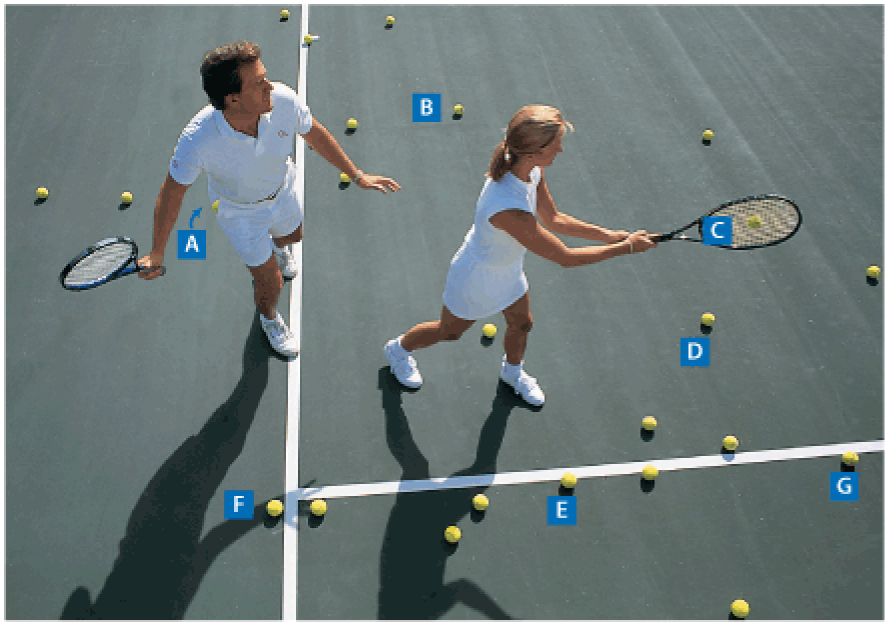


Points *A, B,* and *C* are collinear.

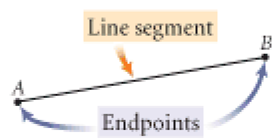
Coplanar means on the same plane.

Points *D, E,* and *F* are coplanar.

Ball A is in the pocket of the man. Ball C is on the woman's racquet. All other balls are on the tennis court. Name three balls that are collinear. Name three balls that are coplanar but not collinear. Name four balls that are not coplanar.



A **line segment** consists of two points called the **endpoints** of the segment and all the points between them that are collinear with the two points.

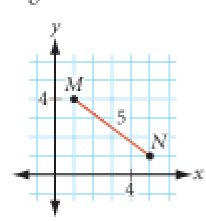


You can write line segment *AB*, using a segment symbol, as $\overline{AB}$ or $\overline{BA}$. There are two ways to write the length of a segment. You can write *AB* = 2 in., meaning the distance from *A* to *B* is 2 inches. You can also use an *m* for "measure" in front of the segment name, and write the distance as $m\overline{AB} = 2$ in. If no measurement units are used for the length of a segment, it is understood that the choice of units is not important or is based on the length of the smallest square in the grid.

Figure A

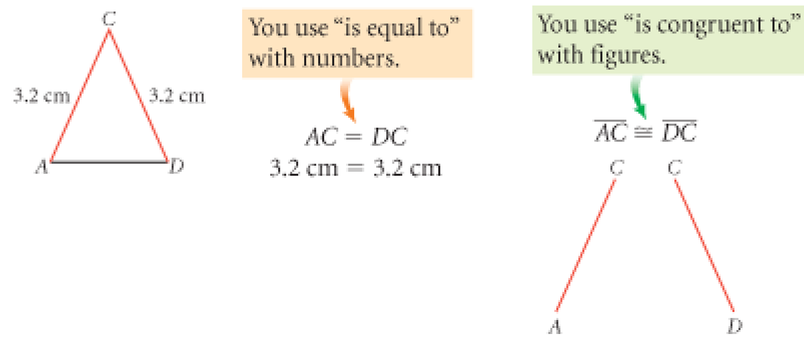
$AB = 2$ in., or $m\overline{AB} = 2$ in.

Figure B

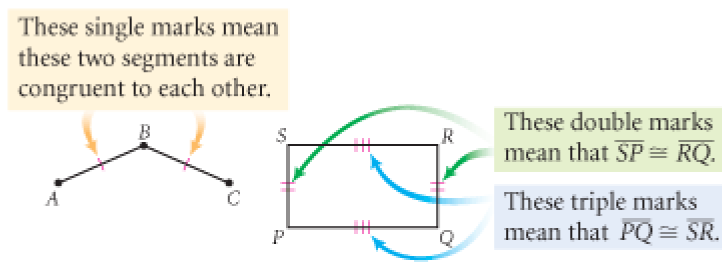


$MN = 5$ units, or $m\overline{MN} = 5$ units

Two segments are **congruent** if and only if they have equal measures, or lengths.



When drawing figures, you show congruent segments by making identical markings.



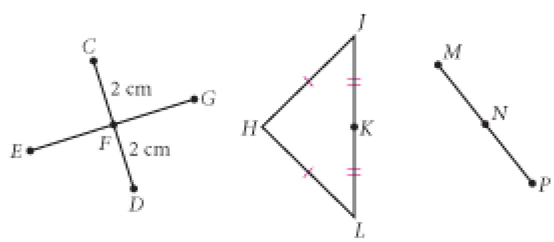
The **midpoint** of a segment is the point on the segment that is the same distance from both endpoints. The midpoint **bisects** the segment, or divides the segment into two congruent segments.

EXAMPLE

Study the diagrams below.

a. Name each midpoint and the segment it bisects.

b. Name all the congruent segments. Use the congruence symbol to write your answers.



▸ Solution

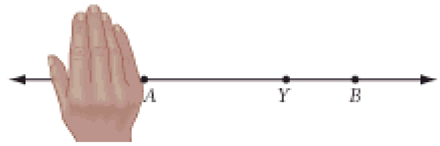
Look carefully at the markings and apply the midpoint definition.

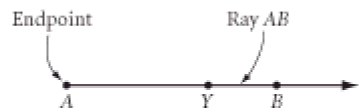
a. $CF = FD$, so F is the midpoint of $\overline{CD}$; $\overline{JK} \cong \overline{KL}$, so K is the midpoint of $\overline{JL}$.

b. $\overline{CF} \cong \overline{FD}$, $\overline{HJ} \cong \overline{HL}$, and $\overline{JK} \cong \overline{KL}$.

Even though $\overline{EF}$ and $\overline{FG}$ appear to have the same length, you cannot assume they are congruent without the markings. The same is true for $\overline{MN}$ and $\overline{NP}$.

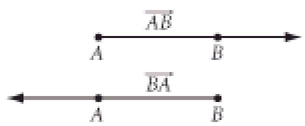
Ray *AB* is the part of $\overleftrightarrow{AB}$ that contains point *A* and all the points on $\overleftrightarrow{AB}$ that are on the same side of point *A* as point *B*. Imagine cutting off all the points to the left of point *A*.

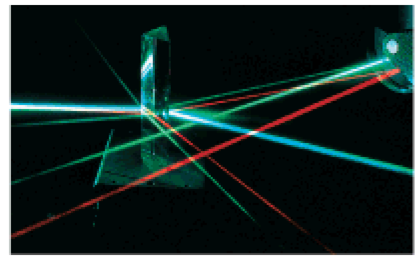




In the figure above, $\overrightarrow{AY}$ and $\overrightarrow{AB}$ are two ways to name the same ray. Note that $\overrightarrow{AB}$ is not the same as $\overrightarrow{BA}$!

A ray begins at a point and extends infinitely in one direction. You need two letters to name a ray. The first letter is the endpoint of the ray, and the second letter is any other point that the ray passes through.





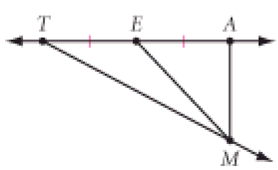
Physical model of a ray: beams of light

Investigation
Mathematical Models

In this lesson, you encountered many new geometry terms. In this investigation you will work as a group to identify models from the real world that represent these terms and to identify how they are represented in diagrams.

Step 1 Look around your classroom and identify examples of each of these terms: point, line, plane, line segment, congruent segments, midpoint of a segment, and ray.

Step 2 Identify examples of these terms in the photograph at right.



Step 3 Identify examples of these terms in the figure above.

Step 4 Explain in your own words what each of these terms means.

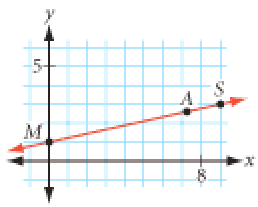
EXERCISES

1. In the photos below identify the physical models that represent a point, segment, plane, collinear points, and coplanar points.

For Exercises 2–4, name each line in two different ways.

2. 

3. 

4. 

For Exercises 5–7, draw two points and label them. Then use a ruler to draw each line. Don’t forget to use arrowheads to show that the line extends indefinitely.

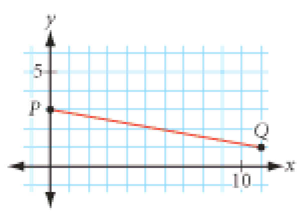
5. $\overleftrightarrow{AB}$

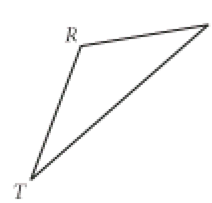
6. $\overleftrightarrow{KL}$

7. $\overleftrightarrow{DE}$ with $D(-3, 0)$ and $E(0, -3)$

For Exercises 8–10, name each line segment.

8. 

9. 

10. 

For Exercises 11 and 12, draw and label each line segment.

11. $\overline{AB}$

12. $\overline{RS}$ with $R(0, 3)$ and $S(-2, 11)$

For Exercises 13 and 14, use your ruler to find the length of each line segment to the nearest tenth of a centimeter. Write your answer in the form $m\overline{AB}$ = __?__ .

13. 

14. 

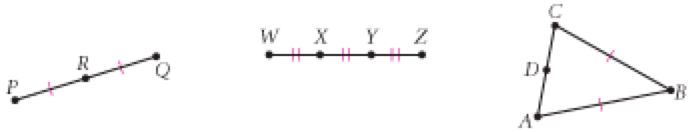
For Exercises 15–17, use your ruler to draw each segment as accurately as you can. Label each segment.

15. $AB = 4.5$ cm

16. $CD = 3$ in.

17. $EF = 24.8$ cm

18. Name each midpoint and the segment it bisects.



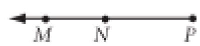
19. Draw two segments that have the same midpoint. Mark your drawing to show congruent segments.

20. Draw and mark a figure in which *M* is the midpoint of $\overline{ST}$, *SP* = *PT*, and T is the midpoint of $\overline{PQ}$.

For Exercises 21–23, name the ray in two different ways.

21.

22.



23.

For Exercises 24–26, draw and label each ray.

24. $\overrightarrow{AB}$

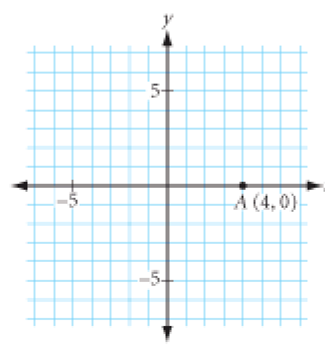
25. $\overrightarrow{YX}$

26. $\overrightarrow{MN}$

27. Draw a plane containing four coplanar points *A, B, C,* and *D,* with exactly three collinear points *A, B,* and *D.*

28. Given two points *A* and *B,* there is only one segment that you can name: $\overline{AB}$. With three collinear points *A, B,* and *C,* there are three different segments that you can name: $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. With five collinear points *A, B, C, D,* and *E,* how many different segments can you name?

For Exercises 29–31, draw axes on graph paper and locate point *A* (4, 0) as shown.



29. Draw $\overline{AB}$ where point *B* has coordinates (2, −6).

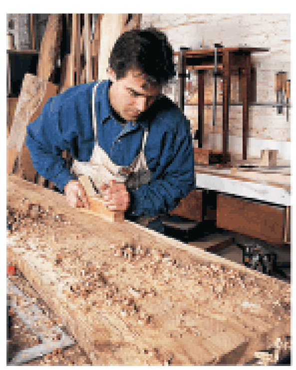
30. Draw $\overrightarrow{OM}$ with endpoint (0, 0) that goes through point *M* (2, 2).

31. Draw $\overleftrightarrow{CD}$ through points *C* (−2, 1) and *D* (−2, −3).

Career CONNECTION

Woodworkers use a tool called a plane to shave a rough wooden surface to create a perfectly smooth planar surface. The smooth board can then be made into a tabletop, a door, or a cabinet.

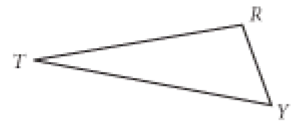
Woodworking is a very precise process. Producing high-quality pieces requires an understanding of lines, planes, and angles as well as careful measurements.



32. If the signs of the coordinates of collinear points $P(-6, -2)$, $Q(-5, 2)$, and $R(-4, 6)$ are reversed, are the three new points still collinear? Draw a picture and explain why.

33. Draw a segment with midpoint $N(-3, 2)$. Label it $\overline{PQ}$.

34. Copy triangle TRY shown at right. Use your ruler to find the midpoint A of side $\overline{TR}$ and the midpoint G of side $\overline{TY}$. Draw $\overline{AG}$.

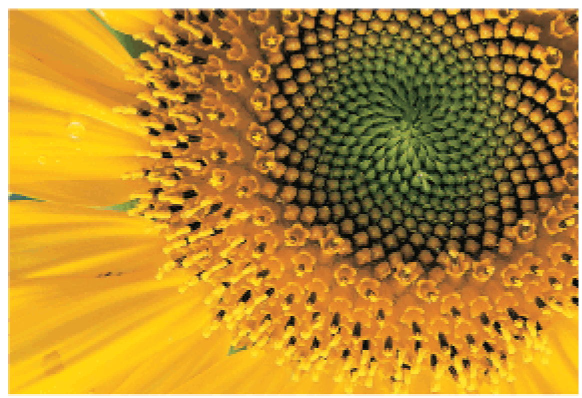


35. Use your ruler to draw a triangle with side lengths 8 cm and 11 cm. Explain your method. Can you draw a second triangle with these two side lengths that looks different from the first?

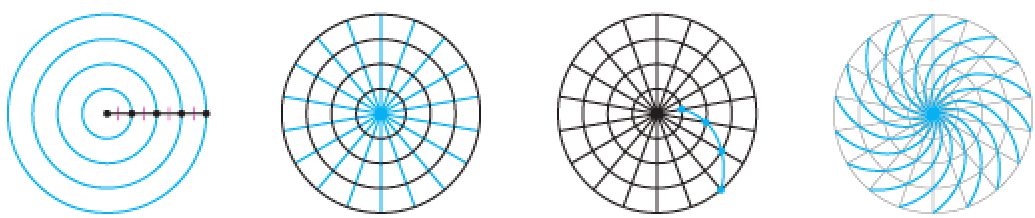
project

SPIRAL DESIGNS

The circle design shown below is used in a variety of cultures to create mosaic decorations. The spiral design may have been inspired by patterns in nature. Notice that the seeds on the sunflower also spiral out from the center.



Create and decorate your own spiral design. Here are the steps to make the spirals. The more circles and radii you draw, the more detailed your design will be.



Step 1 Step 2 Step 3 Step 4

- A completed spiral.
- Coloring or decorations that make the spiral stand out.

[For help, see the **Dynamic Geometry Exploration** Spiral Designs at www.keymath.com/DG .]

keymath.com/DG

USING YOUR ALGEBRA SKILLS 1

Midpoint

A midpoint is the point on a line segment that is the same distance from both endpoints.

You can think of a midpoint as being halfway between two locations. You know how to mark a midpoint. But when the position and location matter, such as in navigation and geography, you can use a coordinate grid and some algebra to find the exact location of the midpoint. You can calculate the coordinates of the midpoint of a segment on a coordinate grid using a formula.

Coordinate Midpoint Property

If (x_1, y_1) and (x_2, y_2) are the coordinates of the endpoints of a segment, then the coordinates of the midpoint are

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

History CONNECTION

Surveyors and mapmakers of ancient Egypt, China, Greece, and Rome used various coordinate systems to locate points. Egyptians made extensive use of square grids and used the first known rectangular coordinates at Saqqara around 2650 B.C.E. By the 17th century, the age of European exploration, the need for accurate maps and the development of easy-to-use algebraic symbols gave rise to modern coordinate geometry. Notice the lines of latitude and longitude in this 17th-century map.

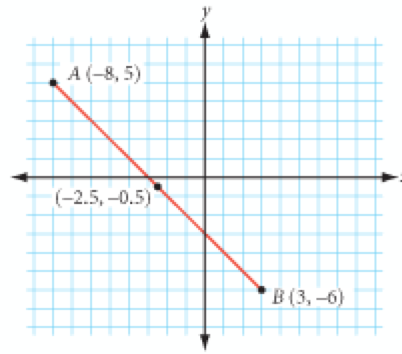
EXAMPLE Segment *AB* has endpoints (−8, 5) and (3, −6). Find the coordinates of the midpoint of $\overline{AB}$.

▸ Solution The midpoint is not on a grid intersection point, so we can use the coordinate midpoint property.

$$= \frac{x_1 + x_2}{2} = \frac{-8+3}{2} = -2.5$$

$$= \frac{y_1 + y_2}{2} = \frac{5+(-6)}{2} = -0.5$$

The midpoint of $\overline{AB}$ is (−2.5, −0.5).



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EXERCISES

For Exercises 1–3, find the coordinates of the midpoint of the segment with each pair of endpoints.

1. (12, −7) and (−6, 15) **2.** (−17, −8) and (−1, 11) **3.** (14, −7) and (−3, 18)

4. One endpoint of a segment is (12, −8). The midpoint is (3, 18). Find the coordinates of the other endpoint.

5. A classmate tells you, "Finding the coordinates of a midpoint is easy. You just find the averages." Is there any truth to it? Explain what you think your classmate means.

6. Find the two points on $\overline{AB}$ that divide the segment into three congruent parts. Point *A* has coordinates (0, 0) and point *B* has coordinates (9, 6). Explain your method.

7. Describe a way to find points that divide a segment into fourths.

8. In each figure below, imagine drawing the diagonals $\overline{AC}$ and $\overline{BD}$

a. Find the midpoint of $\overline{AC}$ and the midpoint of $\overline{BD}$ in each figure.

b. What do you notice about the midpoints?